Appendix to Inflation Differentials between Spain and the EMU: A DSGE Perspective

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Abstract

In this appendix we provide all the symmetric equilibrium conditions of the model, we show how to transform variables to make them stationary, and we present the full set of log-linearized equations. We also report the parameter estimates from estimating the model with the price and quantity variables in levels, and additional posterior second moments of the variables implied by each model

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In this Appendix, we present the following sections. Section 1 presents the symmetric equilibrium conditions, section 2 presents the normalized equilibrium conditions, while section 3 presents the loglinearized model, and the relationship between observable variables in the data and in the model. Section 4 briefly explains how to estimate the model with price and quantity variables in levels, and presents the results from conducting such estimation. Section 5 presents additional posterior second moments of the observable variables implied by the models.

1 Symmetric Equilibrium Conditions

Euler equation

$$\mu_t = \beta E_t \left\{ \mu_{t+1} \frac{R_t P_t}{P_{t+1}} \right\} \tag{1}$$

where μ_t and μ_t^* is the marginal utility of consumption in each country with external habit formation:

$$\mu_t = \frac{1}{C_t - bC_{t-1}} \tag{2}$$

$$\mu_t^* = \frac{1}{C_t^* - bC_{t-1}^*} \tag{3}$$

The risk sharing condition under complete markets delivers:

$$RER = \frac{P_t^*}{P_t} = \frac{\mu_t^*}{\mu_t} \tag{4}$$

1.1 Demand functions

The aggregate consumption demand functions by type of good are:

$$C_t^H = \lambda\gamma \left(\frac{P_t^H}{P_t^T}\right)^{-\nu} \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t \tag{5}$$

$$C_t^F = (1-\lambda)\gamma\xi_t^{F,C} \left(\frac{P_t^F}{P_t^T}\right)^{-\nu} \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t \tag{6}$$

$$C_t^N = (1-\gamma)\xi_t^{N,C} \left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t \tag{7}$$

And, in the foreign country,

$$C_t^{H^*} = (1-\lambda^*)\gamma^* \xi_t^{H^*,C} \left(\frac{P_t^H}{P_t^{T^*}}\right)^{-\nu} \left(\frac{P_t^{T^*}}{P_t^*}\right)^{-\varepsilon} C_t^* \tag{8}$$

$$C_t^{F^*} = \lambda^*\gamma^* \left(\frac{P_t^F}{P_t^{T^*}}\right)^{-\nu} \left(\frac{P_t^{T^*}}{P_t^*}\right)^{-\varepsilon} C_t^* \tag{9}$$

$$C_t^{N^*} = (1-\gamma^*)\xi_t^{N^*,C} \left(\frac{P_t^{N^*}}{P_t^*}\right)^{-\varepsilon} C_t^* \tag{10}$$

Since there is no price discrimination and all prices are set in euros, we have subtituted everywhere the fact that $P_t^H = P_t^{H^*}$ and $P_t^F = P_t^{F^*}$.

1.2 Labor supply

$$L_t^{\varpi} = \mu_t W_t$$

$$(L_t^*)^{\varpi^*} = \mu_t^* W_t^*$$

where total labor is allocated between tradable and nontradable activities:

$$L_t = L_t^T + L_t^N$$

$$L_t^* = L_t^{T^*} + L_t^{N^*}$$

1.3 Technology

The production functions in the two sectors are:

$$y_t^N = X_t Z_t^N L_t^N \tag{11}$$

and

$$y_t^H = X_t Z_t^T L_t^T \tag{12}$$

In the foreign country,

$$y_t^{N^*} = X_t Z_t^{N^*} L_t^{N^*} \tag{13}$$

and

$$y_t^{F^*} = X_t Z_t^{T^*} L_t^{T^*} \tag{14}$$

1.4 Price setting

Denote by $\hat{p}_t^N$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\hat{p}_t^N}{P_t^N} K_t^{N,1} = \frac{\sigma}{(\sigma-1)} K_t^{N,2} \tag{15}$$

where

$$\begin{aligned} K_t^{N,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_N^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N} \left(\Pi^N\right)^{s(1-\varphi_N)}}{\Pi_{t+s}^N} \right)^{1-\sigma} Y_{t+k}^N \\ &= \mu_t Y_t^N + \beta \theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N} \left(\Pi^N\right)^{(1-\varphi_N)}}{\Pi_{t+1}^N} \right]^{1-\sigma} K_{t+1}^{N,1} \right\}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} K_t^{N,2} &= \sum_{k=0}^{\infty} \beta^k \theta_N^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N} \left(\Pi^N\right)^{s(1-\varphi_N)}}{\Pi_{t+s}^N} \right)^{-\sigma} \frac{W_{t+k}}{X_{t+k} Z_{t+k}^N} \frac{P_{t+k}}{P_{t+k}^N} Y_{t+k}^N \\ &= \mu_t \frac{W_t}{X_t Z_t^N} \frac{P_t}{P_t^N} Y_t^N + \beta \theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N} \left(\Pi^N\right)^{(1-\varphi_N)}}{\Pi_{t+1}^N} \right]^{-\sigma} K_{t+1}^{N,2} \right\} \end{aligned} \tag{17}$$

The evolution of the price level of non-tradables is

$$P_t^N \equiv \left[\theta_N \left(P_{t-1}^N \left(\Pi_{t-1}^N\right)^{\varphi_N} \right)^{1-\sigma} + (1-\theta_N) \left(\hat{p}_t^N\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{18}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

Denote by $\hat{p}_t^H$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\hat{p}_t^H}{P_t^H} K_t^{H,1} = \frac{\sigma}{(\sigma-1)} K_t^{H,2} \tag{19}$$

where

$$
\begin{aligned}
K_t^{H,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_H^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H} \left(\Pi^H\right)^{s(1-\varphi_H)}}{\Pi_{t+s}^H} \right)^{1-\sigma} Y_{t+k}^H \qquad (20) \\
&= \mu_t Y_t^H + \beta \theta_H E_t \left\{ \left[\frac{\left(\Pi_t^H\right)^{\varphi_H} \left(\Pi^H\right)^{(1-\varphi_H)}}{\Pi_{t+1}^H} \right]^{1-\sigma} K_{t+1}^{H,1} \right\},
\end{aligned}
$$

and

$$
\begin{aligned}
K_t^{H,2} &= \sum_{k=0}^{\infty} \beta^k \theta_H^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H} \left(\Pi^H\right)^{s(1-\varphi_H)}}{\Pi_{t+s}^H} \right)^{-\sigma} \frac{W_{t+k}}{X_{t+k} Z_{t+k}^H} \frac{P_{t+k}}{P_{t+k}^H} Y_{t+k}^H \\
&= \mu_t \frac{W_t}{X_t Z_t^H} \frac{P_t}{P_t^H} Y_t^H + \beta \theta_H E_t \left\{ \left[\frac{\left(\Pi_t^H\right)^{\varphi_H} \left(\Pi^H\right)^{(1-\varphi_H)}}{\Pi_{t+1}^H} \right]^{-\sigma} K_{t+1}^{H,2} \right\} \qquad (21)
\end{aligned}
$$

The evolution of the price level of non-tradables is

$$
P_t^H \equiv \left[\theta_H \left(P_{t-1}^H \left(\Pi_{t-1}^H\right)^{\varphi_H} \right)^{1-\sigma} + (1-\theta_H) \left(\hat{p}_t^H\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \qquad (22)
$$

where $\Pi_{t-1}^H = \frac{P_{t-1}^H}{P_{t-2}^H}$.

Denote by $\hat{p}_t^{N^*}$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$
\frac{\hat{p}_t^{N^*}}{P_t^{N^*}} K_t^{N^*,1} = \frac{\sigma}{(\sigma-1)} K_t^{N^*,2} \qquad (23)
$$

where

$$
\begin{aligned}
K_t^{N^*,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_{N^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}} \left(\Pi^{N^*}\right)^{s(1-\varphi_{N^*})}}{\Pi_{t+s}^{N^*}} \right)^{1-\sigma} Y_{t+k}^{N^*} \qquad (24) \\
&= \mu_t^* Y_t^{N^*} + \beta \theta_{N^*} E_t \left\{ \left[\frac{\left(\Pi_t^{N^*}\right)^{\varphi_{N^*}} \left(\Pi^{N^*}\right)^{(1-\varphi_{N^*})}}{\Pi_{t+1}^{N^*}} \right]^{1-\sigma} K_{t+1}^{N^*,1} \right\},
\end{aligned}
$$

and

$$
\begin{aligned}
K_t^{N^*,2} &= \sum_{k=0}^{\infty} \beta^k \theta_{N^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}} \left(\Pi^{N^*}\right)^{s(1-\varphi_{N^*})}}{\Pi_{t+s}^{N^*}} \right)^{-\sigma} \frac{W_{t+k}^*}{X_{t+k} Z_{t+k}^{N^*}} \frac{P_{t+k}^*}{P_{t+k}^{N^*}} Y_{t+k}^{N^*} \\
&= \mu_t^* \frac{W_t^*}{X_t Z_t^{N^*}} \frac{P_t^*}{P_t^{N^*}} Y_t^{N^*} + \beta \theta_{N^*} E_t \left\{ \left[\frac{\left(\Pi_t^{N^*}\right)^{\varphi_{N^*}} \left(\Pi^{N^*}\right)^{(1-\varphi_{N^*})}}{\Pi_{t+1}^{N^*}} \right]^{-\sigma} K_{t+1}^{N^*,2} \right\} \quad (25)
\end{aligned}
$$

The evolution of the price level of non-tradables is

$$
P_t^{N^*} \equiv \left[\theta_{N^*} \left(P_{t-1}^{N^*} \left(\Pi_{t-1}^{N^*}\right)^{\varphi_{N^*}} \right)^{1-\sigma} + (1-\theta_{N^*}) \left(\hat{p}_t^{N^*}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (26)
$$

where $\Pi_{t-1}^{N^*} = \frac{P_{t-1}^{N^*}}{P_{t-2}^{N^*}}$.

Denote by $\hat{p}_t^F$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$
\frac{\hat{p}_t^F}{P_t^F} K_t^{F^*,1} = \frac{\sigma}{(\sigma-1)} K_t^{F^*,2} \quad (27)
$$

where

$$
\begin{aligned}
K_t^{F^*,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_{F^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{F}\right)^{\varphi_{F^*}} \left(\Pi^{F}\right)^{s(1-\varphi_{F^*})}}{\Pi_{t+s}^{F}} \right)^{1-\sigma} Y_{t+k}^{F^*} \quad (28) \\
&= \mu_t^* Y_t^{F^*} + \beta \theta_{F^*} E_t \left\{ \left[\frac{\left(\Pi_t^{F}\right)^{\varphi_{F^*}} \left(\Pi^{F}\right)^{(1-\varphi_{F^*})}}{\Pi_{t+1}^{F}} \right]^{1-\sigma} K_{t+1}^{F^*,1} \right\},
\end{aligned}
$$

and

$$
\begin{aligned}
K_t^{F^*,2} &= \sum_{k=0}^{\infty} \beta^k \theta_{F^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{F}\right)^{\varphi_{F^*}} \left(\Pi^{F}\right)^{s(1-\varphi_{F^*})}}{\Pi_{t+s}^{F}} \right)^{-\sigma} \frac{W_{t+k}^*}{X_{t+k} Z_{t+k}^{F^*}} \frac{P_{t+k}^*}{P_{t+k}^{F}} Y_{t+k}^{F^*} \\
&= \mu_t^* \frac{W_t^*}{X_t Z_t^{F^*}} \frac{P_t^*}{P_t^{F}} Y_t^{F^*} + \beta \theta_{F^*} E_t \left\{ \left[\frac{\left(\Pi_t^{F}\right)^{\varphi_{F^*}} \left(\Pi^{F}\right)^{(1-\varphi_{F^*})}}{\Pi_{t+1}^{F}} \right]^{-\sigma} K_{t+1}^{F^*,2} \right\} \quad (29)
\end{aligned}
$$

The evolution of the price level of non-tradables is

$$
P_t^{F} \equiv \left[\theta_{F^*} \left(P_{t-1}^{F} \left(\Pi_{t-1}^{F}\right)^{\varphi_{F^*}} \right)^{1-\sigma} + (1-\theta_{F^*}) \left(\hat{p}_t^{F}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (30)
$$

where $\Pi_{t-1}^{F} = \frac{P_{t-1}^{F}}{P_{t-2}^{F}}$.

1.5 Relevant price indices

The Spanish consumer price inflation is given by

$$P_t^{1-\varepsilon} = \left[\gamma \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma)\xi_t^{N,C} \left(P_t^N\right)^{1-\varepsilon}\right] \tag{31}$$

where:

$$\left(P_t^T\right)^{1-\nu} = \left[\lambda \left(P_t^H\right)^{1-\nu} + (1-\lambda)\xi_t^{F,C} \left(P_t^F\right)^{1-\nu}\right] \tag{32}$$

The rest of euro area aggregates are given by:

$$\left(P_t^*\right)^{1-\varepsilon} = \left[\gamma^* \left(P_t^{T^*}\right)^{1-\varepsilon} + (1-\gamma^*)\xi_t^{N^*,C} \left(P_t^{N^*}\right)^{1-\varepsilon}\right] \tag{33}$$

where

$$\left(P_t^{T^*}\right)^{1-\nu} = \left[(1-\lambda^*)\xi_t^{H^*,C} \left(P_t^{H^*}\right)^{1-\nu} + \lambda^* \left(P_t^{F^*}\right)^{1-\nu}\right]. \tag{34}$$

The European Central Bank targets the euro-area wide CPI, which is assumed to be an harmonic mean of the price levels of Spain and the rest of the euro area:

$$P_t^{EMU} = P_t^s \left(P_t^*\right)^{1-s} \tag{35}$$

using the country sizes as weights.

The relative price variables move as follows:

$$T_t^N = \frac{P_t^N}{P_t} \tag{36}$$

$$T_t^H = \frac{P_t^H}{P_t} \tag{37}$$

$$T_t^F = \frac{P_t^F}{P_t} \tag{38}$$

$$T_t^{N^*} = \frac{P_t^{N^*}}{P_t} \tag{39}$$

$$T_t^{F^*} = \frac{P_t^F}{P_t^*} \tag{40}$$

$$T_t^{H^*} = \frac{P_t^H}{P_t^*} \tag{41}$$

and the real exchange rate moves as:

$$RER_t = \frac{P_t^*}{P_t} \tag{42}$$

1.6 Market Clearing

The market clearing for the nontradable goods sector at home is

$$y_t^N = Y_t^N = C_t^N + G_t^N \tag{43}$$

The market clearing condition in the tradable goods sector at home is

$$y_t^H = Y_t^H = C_t^H + C_t^{H^*} + G_t^T \tag{44}$$

Aggregate real GDP aggregates tradable and nontradable goods using the appropiate relative prices:

$$Y_t = \frac{P_t^H}{P_t} Y_t^H + \frac{P_t^N}{P_t} Y_t^N \tag{45}$$

For the foreign country, these expressions become:

$$y_t^{N^*} = Y_t^{N^*} = C_t^{N^*} + G_t^{N^*}, \tag{46}$$

$$y_t^{F^*} = Y_t^{F^*} = C_t^F + C_t^{F^*} + G_t^{T^*}, \tag{47}$$

and:

$$Y_t^* = \frac{P_t^{F^*}}{P_t^*} Y_t^{F^*} + \frac{P_t^{N^*}}{P_t^*} Y_t^{N^*}. \tag{48}$$

1.7 Monetary Policy

In order to abstract from fiscal policy considerations, it is assumed that government spending in the two areas is financed through lump sum taxes. Monetary policy is conducted by the ECB with a Taylor rule that only targets the EMU CPI:

$$R_t = R_{t-1}^{\rho_r} \left(\frac{P_t^{EMU} / P_{t-1}^{EMU}}{\bar{\Pi}} \right)^{(1-\rho_r)\gamma_\pi} \exp(\varepsilon_t^m). \tag{49}$$

where ε_t^m is an iid monetary policy shock.

1.8 Shocks Processes

$$X_t = (1+x)^t X_0 \tag{50}$$

$$\begin{aligned} Z_t^N &= (1+\alpha^N)^t \tilde{Z}_t^N \\ \log(\tilde{Z}_t^N) &= \rho^{Z,N} \log(\tilde{Z}_{t-1}^N) + \varepsilon_t^{Z,N} \end{aligned} \tag{51}$$

$$\begin{aligned} Z_t^T &= (1+\alpha^T)^t \tilde{Z}_t^T \\ \log(\tilde{Z}_t^T) &= \rho^{Z,T} \log(\tilde{Z}_{t-1}^T) + \varepsilon_t^{Z,T} + \varepsilon_t^Z \end{aligned} \tag{52}$$

$$\begin{aligned} G_t^N &= (1+x)^t (1+\alpha^N)^t \tilde{G}_t^N \\ \log(\tilde{G}_t^N) &= \rho^{G,N} \log(\tilde{G}_t^N) + \varepsilon_t^{G,N} \end{aligned} \tag{53}$$

$$\begin{aligned} G_t^T &= (1+x)^t (1+\alpha^T)^t \tilde{G}_t^T \\ \log(\tilde{G}_t^T) &= \rho^{G,T} \log(\tilde{G}_t^N) + \varepsilon_t^{G,T} \end{aligned} \tag{54}$$

$$\begin{aligned} Z_t^{N^*} &= (1+\alpha^{N^*})^t \tilde{Z}_t^{N^*} \\ \log(\tilde{Z}_t^{N^*}) &= \rho^{Z,N^*} \log(\tilde{Z}_{t-1}^{N^*}) + \varepsilon_t^{Z,N^*} \end{aligned} \tag{55}$$

$$\begin{aligned} Z_t^{T^*} &= (1+\alpha^{T^*})^t \tilde{Z}_t^{T^*} \\ \log(\tilde{Z}_t^{T^*}) &= \rho^{Z,T^*} \log(\tilde{Z}_{t-1}^{T^*}) + \varepsilon_t^{Z,T^*} + \varepsilon_t^Z \end{aligned} \tag{56}$$

$$\begin{aligned} G_t^{N^*} &= (1+x)^t(1+\alpha^{N^*})^t\tilde{G}_t^{N^*} \\ \log(\tilde{G}_t^{N^*}) &= \rho^{G,N^*}\log(\tilde{G}_t^{N^*}) + \varepsilon_t^{G,N^*} \end{aligned} \tag{57}$$

$$\begin{aligned} G_t^{T^*} &= (1+x)^t(1+\alpha^{T^*})^t\tilde{G}_t^{T^*} \\ \log(\tilde{G}_t^{T^*}) &= \rho^{G,T^*}\log(\tilde{G}_t^{T^*}) + \varepsilon_t^{G,T^*} \end{aligned} \tag{58}$$

2 Normalized Equilibrium Conditions

Here we list again all the equilibrium conditions of the model when technology grows at different rates in each sector, and therefore inflation rates and real growth rates are different in each sector. As we go along we define the particular normalization each variable needs.

2.1 Technology

The production functions in the two sectors are:

$$y_t^N = \frac{y_t^N}{[(1+\alpha^N)(1+x)]^t} = X_0\tilde{Z}_t^N L_t^N \tag{59}$$

and

$$\tilde{y}_t^H = \frac{y_t^H}{[(1+\alpha^T)(1+x)]^t} = X_0\tilde{Z}_t^T L_t^T \tag{60}$$

and

$$\tilde{y}_t^{N^*} = \frac{y_t^{N^*}}{[(1+\alpha^{N^*})(1+x)]^t} = X_0\tilde{Z}_t^{N^*} L_t^{N^*} \tag{61}$$

and

$$\tilde{y}_t^{F^*} = \frac{y_t^{F^*}}{[(1+\alpha^{T^*})(1+x)]^t} = X_0\tilde{Z}_t^{T^*} L_t^{T^*} \tag{62}$$

2.2 Relevant price indices

Next, from the pricing equations in the steady state we obtain the following relationships:

$$Z^T P^H = Z^N P^N$$

and

$$Z^{T^*} P^{F^*} = Z^{N^*} P^{N^*}$$

These relationships also hold for the normalized variables:

$$\tilde{Z}^T \tilde{P}^H = \tilde{Z}^N \tilde{P}^N$$

and

$$\tilde{Z}^{T^*} \tilde{P}^{F^*} = \tilde{Z}^{N^*} \tilde{P}^{N^*}$$

Hence, in each country and sector, the differential growth rate of technology also determines the inflation differential.

We normalize price levels as follows:

$$\begin{aligned}
\tilde{P}_t^H &= \frac{P_t^H (1+\alpha^T)^t}{\Pi^t} \\
\tilde{P}_t^N &= \frac{P_t^N (1+\alpha^N)^t}{\Pi^t} \\
\tilde{P}_t^{F^*} &= \frac{P_t^{F^*} (1+\alpha^{T^*})^t}{\Pi^t} \\
\tilde{P}_t^{N^*} &= \frac{P_t^{N^*} (1+\alpha^{N^*})^t}{\Pi^t}
\end{aligned}$$

Then, if there were no trend growth rates of technology, all prices would grow at a rate Π, which is the inflation target of the ECB.

Then, the price levels are given by

$$\left(\tilde{P}_t^T\right)^{1-\nu} = \left[\lambda \left(\tilde{P}_t^H\right)^{1-\nu} + (1-\lambda)\tilde{\xi}_t^{F,C} \left(\tilde{P}_t^{F^*}\right)^{1-\nu}\right] \tag{63}$$

where $\tilde{\xi}_t^{F,C} = \xi_t^{F,C} \left[\frac{(1+\alpha^T)}{(1+\alpha^{T^*})}\right]^{(1-\nu)t}$, and $\tilde{P}_t^T = P_t^T \frac{(1+\alpha^T)^t}{\Pi^t}$. Note that the preference shock absorbs the possible difference in the trends of the tradable good technology shocks across countries.

For the aggregate CPI:

$$\tilde{P}_t^{1-\varepsilon} = \left[\gamma\left(\tilde{P}_t^T\right)^{1-\varepsilon} + (1-\gamma)\tilde{\xi}_t^{N,C}\left(\tilde{P}_t^N\right)^{1-\varepsilon}\right] \tag{64}$$

where $\tilde{P}_t = \frac{P_t(1+\alpha^T)^t}{\bar{\Pi}^t}$, and $\tilde{\xi}_t^{N,C} = \xi_t^{N,C}\left[\frac{(1+\alpha^T)}{(1+\alpha^N)}\right]^{(1-\varepsilon)t}$. In this case the preference shock allows the normalization of price levels across sectors that can have different trends.

The rest of euro area aggregates are given by:

$$\left(\tilde{P}_t^*\right)^{1-\varepsilon} = \left[\gamma\left(\tilde{P}_t^{T^*}\right)^{1-\varepsilon} + (1-\gamma)\tilde{\xi}_t^{N^*,C}\left(\tilde{P}_t^{N^*}\right)^{1-\varepsilon}\right] \tag{65}$$

where $\tilde{P}_t^* = \frac{P_t^*(1+\alpha^{T^*})^t}{\bar{\Pi}^t}$, and $\tilde{\xi}_t^{N^*,C} = \xi_t^{N^*,C}\left[\frac{(1+\alpha^{T^*})}{(1+\alpha^{N^*})}\right]^{(1-\varepsilon)t}$, and

$$\left(\tilde{P}_t^{T^*}\right)^{1-\nu} = \left[(1-\lambda)\tilde{\xi}_t^{H^*,C}\left(\tilde{P}_t^{H^*}\right)^{1-\nu} + \lambda\left(\tilde{P}_t^{F^*}\right)^{1-\nu}\right]. \tag{66}$$

where $\tilde{\xi}_t^{H^*,C} = \xi_t^{H^*,C}\left(\frac{1+\alpha^{T^*}}{1+\alpha^T}\right)^{(1-\nu)t}$ and $\tilde{P}_t^{T^*} = \frac{P_t^{T^*}(1+\alpha^{T^*})^t}{\bar{\Pi}^t}$.

The European Central Bank targets the euro-area wide CPI, which is assumed to be an harmonic mean of the price levels of Spain and the rest of the euro area:

$$P_t^{EMU}\frac{\left(1+\alpha^T\right)^{ts}\left(1+\alpha^{T^*}\right)^{t(1-s)}}{\Pi^t} = \tilde{P}_t^{EMU} = \tilde{P}_t^s\left(\tilde{P}_t^*\right)^{1-s} \tag{67}$$

using the country sizes as weights.

2.3 Market Clearing

The market clearing for the nontradable goods sector at home is

$$\tilde{Y}_t^N = \tilde{C}_t^N + \tilde{G}_t^N \tag{68}$$

where $\tilde{C}_t^N = \frac{C_t^N}{[(1+\alpha^N)(1+x)]^t}$ and $\tilde{G}_t^N = \frac{G_t^N}{[(1+\alpha^N)(1+x)]^t}$.

The market clearing condition in the tradable goods sector at home is

$$\tilde{Y}_t^H = \tilde{C}_t^H + \tilde{C}_t^{H^*} + \tilde{G}_t^T \tag{69}$$

where $\tilde{C}_t^H = \frac{C_t^H}{[(1+\alpha^T)(1+x)]^t}$, $\tilde{C}_t^{H^*} = \frac{C_t^{H^*}}{[(1+\alpha^T)(1+x)]^t}$, and $\tilde{G}_t^T = \frac{G_t^T}{[(1+\alpha^T)(1+x)]^t}$.

Aggregate real GDP aggregates tradable and nontradable goods using the appropiate relative prices:

$$\tilde{Y}_t = \frac{\tilde{P}_t^H}{\tilde{P}_t}\tilde{Y}_t^H + \frac{\tilde{P}_t^N}{\tilde{P}_t}\tilde{Y}_t^N \tag{70}$$

where $\tilde{y}_t = \frac{y_t}{[(1+\alpha^T)(1+x)]^t}$.

For the foreign country, these expressions become:

$$\tilde{Y}_t^{N^*} = \tilde{C}_t^{N^*} + \tilde{G}_t^{N^*}, \tag{71}$$

where $\tilde{C}_t^{N^*} = \frac{C_t^{N^*}}{\left[(1+\alpha^{N^*})(1+x)\right]^t}$ and $\tilde{G}_t^{N^*} = \frac{G_t^{N^*}}{\left[(1+\alpha^{N^*})(1+x)\right]^t}$.

$$\tilde{Y}_t^{F^*} = \tilde{C}_t^F + \tilde{C}_t^{F^*} + \tilde{G}_t^{T^*}, \tag{72}$$

where $\tilde{C}_t^F = \frac{C_t^F}{\left[(1+\alpha^{T^*})(1+x)\right]^t}$, $\tilde{C}_t^{F^*} = \frac{C_t^{F^*}}{\left[(1+\alpha^{T^*})(1+x)\right]^t}$, and $\tilde{G}_t^{T^*} = \frac{G_t^{T^*}}{\left[(1+\alpha^{T^*})(1+x)\right]^t}$.

Finally:

$$\tilde{Y}_t^* = \frac{\tilde{P}_t^F}{\tilde{P}_t^*}\tilde{Y}_t^{F^*} + \frac{\tilde{P}_t^{N^*}}{\tilde{P}_t^*}\tilde{Y}_t^{N^*}. \tag{73}$$

2.4 Demand functions

The aggregate consumption demand functions by type of good are:

$$\tilde{C}_t^H = \lambda\gamma\left(\frac{\tilde{P}_t^H}{\tilde{P}_t^T}\right)^{-\nu}\left(\frac{\tilde{P}_t^T}{\tilde{P}_t}\right)^{-\varepsilon}\tilde{C}_t \tag{74}$$

where $\tilde{C}_t = \frac{C_t}{[(1+\alpha^T)(1+x)]^t}$.

$$\tilde{C}_t^F = (1-\lambda)\gamma\tilde{\xi}_t^{F,C}\left(\frac{\tilde{P}_t^F}{\tilde{P}_t^T}\right)^{-\nu}\left(\frac{\tilde{P}_t^T}{\tilde{P}_t}\right)^{-\varepsilon}\tilde{C}_t \tag{75}$$

$$\tilde{C}_t^N = (1-\gamma)\tilde{\xi}_t^{N,C}\left(\frac{\tilde{P}_t^N}{\tilde{P}_t}\right)^{-\varepsilon}\tilde{C}_t \tag{76}$$

$$\tilde{C}_t^{H^*} = (1-\lambda^*)\gamma^*\tilde{\xi}_t^{H^*,C}\left(\frac{\tilde{P}_t^H}{P_t^{T^*}}\right)^{-\nu}\left(\frac{\tilde{P}_t^{T^*}}{\tilde{P}_t^*}\right)^{-\varepsilon}\tilde{C}_t^* \tag{77}$$

where $\tilde{C}_t^* = \frac{C_t^*}{\left[(1+\alpha^{T^*})(1+x)\right]^t}$.

$$\tilde{C}_t^{F^*} = \lambda^* \gamma^* \left(\frac{\tilde{P}_t^F}{\tilde{P}_t^{T^*}} \right)^{-\nu} \left(\frac{\tilde{P}_t^{T^*}}{\tilde{P}_t^*} \right)^{-\varepsilon} \tilde{C}_t^* \tag{78}$$

$$\tilde{C}_t^{N^*} = (1-\gamma^*) \tilde{\xi}_t^{N^*,C} \left(\frac{\tilde{P}_t^{N^*}}{\tilde{P}_t^*} \right)^{-\varepsilon} \tilde{C}_t^* \tag{79}$$

2.5 Euler equation and risk sharing

Euler equation and risk sharing:

$$\tilde{\mu}_t = \beta E_t \left\{ \frac{\tilde{\mu}_{t+1}}{(1+x)} \frac{R_t \tilde{P}_t}{\tilde{P}_{t+1} \bar{\Pi}} \right\} \tag{80}$$

where $\tilde{\mu}_t = \mu_t \left[(1+x)(1+\alpha^T)\right]^t$. Then

$$\tilde{\mu}_t = \frac{1}{\tilde{C}_t - \frac{b}{[(1+x)(1+\alpha^T)]} \tilde{C}_{t-1}} \tag{81}$$

$$\tilde{\mu}_t^* = \frac{1}{\tilde{C}_t^* - \frac{b^*}{\left[(1+x)(1+\alpha^{T^*})\right]} \tilde{C}_{t-1}^*} \tag{82}$$

where $\tilde{\mu}_t^* = \mu_t^* \left[(1+x)(1+\alpha^{T^*})\right]^t$. Finally,

$$\widetilde{RER}_t = \frac{P_t^*}{P_t} \left(\frac{1+\alpha^{T^*}}{1+\alpha^T} \right)^t \tag{83}$$

$$= \frac{\tilde{P}_t^*}{\tilde{P}_t} = \frac{\tilde{\mu}_t^*}{\tilde{\mu}_t} \tag{84}$$

Hence, in this version of the model, trends in the real exchange rate are entirely due to different productivity trends across sectors and countries

$$RER_t = \left(\frac{1+\alpha^T}{1+\alpha^{T^*}} \right)^t \widetilde{RER}_t \tag{85}$$

2.6 Labor supply

$$L_t^{\varpi} = \tilde{\mu}_t \tilde{W}_t \tag{86}$$

$$(L_t^*)^{\varpi^*} = \tilde{\mu}_t^* \tilde{W}_t^* \tag{87}$$

where $\tilde{W}_t = \frac{W_t}{[(1+x)(1+\alpha^T)]^t}$, and $\tilde{W}_t^* = \frac{W_t^*}{\left[(1+x)(1+\alpha^{T^*})\right]^t}$, and total labor is allocated between tradable and nontradable activities:

$$L_t = L_t^T + L_t^N \tag{88}$$

$$L_t^* = L_t^{T^*} + L_t^{N^*} \tag{89}$$

2.7 Price setting

Denote by $\hat{p}_t^N$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\breve{p}_t^N}{\tilde{P}_t^N} \tilde{K}_t^{N,1} = \frac{\sigma}{(\sigma-1)} \tilde{K}_t^{N,2} \tag{90}$$

where $\breve{p}_t^N = \frac{\hat{p}_t^N (1+\alpha^N)^t}{\Pi^t}$, and $\tilde{P}_t^N = \frac{P_t^N (1+\alpha^N)^t}{\Pi^t}$, and

$$\begin{aligned} \tilde{K}_t^{N,1} &= \left(\frac{1+\alpha^T}{1+\alpha^N}\right)^t K_t^{N,1} = E_t \sum_{k=0}^{\infty} \beta^k \theta_N^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N} \left(\Pi^N\right)^{s(1-\varphi_N)}}{\Pi_{t+s}^N} \right)^{1-\sigma} Y_{t+k}^N \\ &= \tilde{\mu}_t \tilde{Y}_t^N + \beta\theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N} \left(\Pi^N\right)^{(1-\varphi_N)}}{\Pi_{t+1}^N} \right]^{1-\sigma} \tilde{K}_{t+1}^{N,1} \left(\frac{1+\alpha^N}{1+\alpha^T}\right) \right\}, \end{aligned} \tag{91}$$

and

$$\begin{aligned} \tilde{K}_t^{N,2} &= \left(\frac{1+\alpha^T}{1+\alpha^N}\right)^t K_t^{N,2} = \sum_{k=0}^{\infty} \beta^k \theta_N^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N} \left(\Pi^N\right)^{s(1-\varphi_N)}}{\Pi_{t+s}^N} \right)^{-\sigma} \frac{W_{t+k}}{X_{t+k} Z_{t+k}^N} \frac{P_{t+k}}{P_{t+k}^N} Y_{t+k}^N \\ &= \tilde{\mu}_t \tilde{W}_t \frac{\tilde{P}_t}{\tilde{P}_t^N} \frac{\tilde{Y}_t^N}{X_0 \tilde{Z}_t^N} + \beta\theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N} \left(\Pi^N\right)^{(1-\varphi_N)}}{\Pi_{t+1}^N} \right]^{-\sigma} \tilde{K}_{t+1}^{N,2} \left(\frac{1+\alpha^N}{1+\alpha^T}\right) \right\} \end{aligned} \tag{92}$$

The evolution of the price level of non-tradables is

$$1 \equiv \left[\theta_N \left(\frac{\left(\Pi_{t-1}^N\right)^{\varphi_N}\left(\Pi^N\right)^{(1-\varphi_N)}}{\Pi_t^N}\right)^{1-\sigma} + (1-\theta_N)\left(\frac{\check{p}_t^N}{\tilde{P}_t^N}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{93}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

Denote by $\hat{p}_t^H$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\check{p}_t^H}{\tilde{P}_t^H} K_t^{H,1} = \frac{\sigma}{(\sigma-1)} K_t^{H,2} \tag{94}$$

where $\check{p}_t^H = \frac{\hat{p}_t^H (1+\alpha^T)^t}{\Pi^t}$, and $\tilde{P}_t^H = \frac{P_t^H (1+\alpha^T)^t}{\Pi^t}$,

$$\begin{aligned} K_t^{H,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_H^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H}\left(\Pi^H\right)^{s(1-\varphi_H)}}{\Pi_{t+s}^H}\right)^{1-\sigma} Y_{t+k}^H \\ &= \tilde{\mu}_t \tilde{Y}_t^H + \beta\theta_H E_t \left\{\left[\frac{\left(\Pi_t^H\right)^{\varphi_H}\left(\Pi^H\right)^{(1-\varphi_H)}}{\Pi_{t+1}^H}\right]^{1-\sigma} K_{t+1}^{H,1}\right\}, \end{aligned} \tag{95}$$

and

$$\begin{aligned} K_t^{H,2} &= \sum_{k=0}^{\infty} \beta^k \theta_H^k \mu_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H}\left(\Pi^H\right)^{s(1-\varphi_H)}}{\Pi_{t+s}^H}\right)^{-\sigma} \frac{W_{t+k}}{X_{t+k} Z_{t+k}^T} \frac{P_{t+k}}{P_{t+k}^H} Y_{t+k}^H \\ &= \frac{\tilde{P}_t}{\tilde{P}_t^H} \tilde{W}_t \tilde{\mu}_t \frac{\tilde{Y}_t^H}{X_0 \tilde{Z}_t^T} + \beta\theta_H E_t \left\{\left[\frac{\left(\Pi_t^H\right)^{\varphi_H}\left(\Pi^H\right)^{(1-\varphi_H)}}{\Pi_{t+1}^H}\right]^{-\sigma} K_{t+1}^{H,2}\right\} \end{aligned} \tag{96}$$

The evolution of the price level of tradables is

$$1 \equiv \left[\theta_H \left(\frac{\left(\Pi_{t-1}^H\right)^{\varphi_H}\left(\Pi^H\right)^{(1-\varphi_H)}}{\Pi_t^H}\right)^{1-\sigma} + (1-\theta_H)\left(\frac{\check{p}_t^H}{\tilde{P}_t^H}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{97}$$

where $\Pi_{t-1}^H = \frac{P_{t-1}^H}{P_{t-2}^H}$.

Denote by $\hat{p}_t^{N*}$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\check{p}_t^{N^*}}{\tilde{P}_t^{N^*}}\tilde{K}_t^{N^*,1} = \frac{\sigma}{(\sigma-1)}\tilde{K}_t^{N^*,2} \tag{98}$$

where $\check{p}_t^{N^*} = \frac{\hat{p}_t^{N^*}(1+\alpha^{N^*})^t}{\Pi^t}$, and $\tilde{P}_t^{N^*} = \frac{P_t^{N^*}(1+\alpha^{N^*})^t}{\Pi^t}$,

$$\begin{aligned} \tilde{K}_t^{N^*,1} &= \left(\frac{1+\alpha^{T^*}}{1+\alpha^{N^*}}\right)^t K_t^{N^*,1} = \qquad (99) \\ &= E_t\sum_{k=0}^{\infty}\beta^k\theta_{N^*}^k\mu_{t+k}^*\left(\prod_{s=1}^{k}\frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}\left(\Pi^{N^*}\right)^{s(1-\varphi_{N^*})}}{\Pi_{t+s}^{N^*}}\right)^{1-\sigma} Y_{t+k}^{N^*} \\ &= \tilde{\mu}_t^*\tilde{Y}_t^{N^*} + \beta\theta_{N^*}E_t\left\{\left[\frac{\left(\Pi_t^{N^*}\right)^{\varphi_{N^*}}\left(\Pi^{N^*}\right)^{(1-\varphi_{N^*})}}{\Pi_{t+1}^{N^*}}\right]^{1-\sigma}\tilde{K}_{t+1}^{N^*,1}\left(\frac{1+\alpha^{N^*}}{1+\alpha^{T^*}}\right)\right\}, \end{aligned}$$

and

$$\begin{aligned} \tilde{K}_t^{N^*,2} &= \left(\frac{1+\alpha^{T^*}}{1+\alpha^{N^*}}\right)^t K_t^{N^*,2} = \qquad (100) \\ &= \sum_{k=0}^{\infty}\beta^k\theta_{N^*}^k\mu_{t+k}^*\left(\prod_{s=1}^{k}\frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}\left(\Pi^{N^*}\right)^{s(1-\varphi_{N^*})}}{\Pi_{t+s}^{N^*}}\right)^{-\sigma}\frac{W_{t+k}^*}{X_{t+k}Z_{t+k}^{N^*}}\frac{P_{t+k}^*}{P_{t+k}^{N^*}}Y_{t+k}^{N^*} \\ &= \frac{\tilde{P}_t^*}{\tilde{P}_t^{N^*}}\tilde{W}_t^*\tilde{\mu}_t^*\frac{\tilde{Y}_t^{N^*}}{X_0\tilde{Z}_t^{N^*}} + \beta\theta_{N^*}E_t\left\{\left[\frac{\left(\Pi_t^{N^*}\right)^{\varphi_{N^*}}\left(\Pi^{N^*}\right)^{(1-\varphi_{N^*})}}{\Pi_{t+1}^{N^*}}\right]^{-\sigma}\tilde{K}_{t+1}^{N^*,2}\left(\frac{1+\alpha^{N^*}}{1+\alpha^{T^*}}\right)\right\} \end{aligned}$$

The evolution of the price level of non-tradables is

$$1 \equiv \left[\theta_{N^*}\left(\frac{\left(\Pi_{t-1}^{N^*}\right)^{\varphi_{N^*}}\left(\Pi^{N^*}\right)^{(1-\varphi_{N^*})}}{\Pi_t^{N^*}}\right)^{1-\sigma} + (1-\theta_{N^*})\left(\frac{\check{p}_t^{N^*}}{\tilde{P}_t^{N^*}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{101}$$

where $\Pi_{t-1}^{N^*} = \frac{P_{t-1}^{N^*}}{P_{t-2}^{N^*}}$.

Denote by $\hat{p}_t^F$ the optimal price set by those who receive the Calvo lottery. The optimal price is given by:

$$\frac{\check{p}_t^F}{\tilde{P}_t^F}K_t^{F^*,1} = \frac{\sigma}{(\sigma-1)}K_t^{F^*,2} \tag{102}$$

where $\check{p}_t^F = \frac{\hat{p}_t^F(1+\alpha^{T^*})^t}{\Pi^t}$, and $\tilde{P}_t^F = \frac{P_t^F(1+\alpha^{T^*})^t}{\Pi^t}$,

$$\begin{aligned} K_t^{F^*,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_{F^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^F\right)^{\varphi_{F^*}} \left(\Pi^F\right)^{s(1-\varphi_{F^*})}}{\Pi_{t+s}^F} \right)^{1-\sigma} Y_{t+k}^{F^*} \\ &= \tilde{\mu}_t^* \tilde{Y}_t^{F^*} + \beta\theta_{F^*} E_t \left\{ \left[\frac{\left(\Pi_t^F\right)^{\varphi_{F^*}} \left(\Pi^F\right)^{(1-\varphi_{F^*})}}{\Pi_{t+1}^F} \right]^{1-\sigma} K_{t+1}^{F^*,1} \right\}, \end{aligned} \tag{103}$$

and

$$\begin{aligned} K_t^{F^*,2} &= \sum_{k=0}^{\infty} \beta^k \theta_{F^*}^k \mu_{t+k}^* \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^F\right)^{\varphi_{F^*}} \left(\Pi^F\right)^{s(1-\varphi_{F^*})}}{\Pi_{t+s}^{F^*}} \right)^{-\sigma} \frac{MC_{t+k}^{F^*}}{P_{t+k}^{F^*}} \frac{P_{t+k}^*}{P_{t+k}^F} Y_{t+k}^{F^*} \\ &= \frac{\tilde{P}_t^*}{\tilde{P}_t^F} \tilde{W}_t^* \tilde{\mu}_t^* \frac{\tilde{Y}_t^{F^*}}{X_0 \tilde{Z}_t^{T^*}} + \beta\theta_{F^*} E_t \left\{ \left[\frac{\left(\Pi_t^F\right)^{\varphi_{F^*}} \left(\Pi^F\right)^{(1-\varphi_{F^*})}}{\Pi_{t+1}^F} \right]^{-\sigma} K_{t+1}^{F^*,2} \right\} \end{aligned} \tag{104}$$

The evolution of the price level of non-tradables is

$$1 \equiv \left[\theta_{F^*} \left(\frac{\left(\Pi_{t-1}^F\right)^{\varphi_{F^*}} \left(\Pi^F\right)^{(1-\varphi_{F^*})}}{\Pi_t^F} \right)^{1-\sigma} + (1-\theta_{F^*}) \left(\frac{\check{p}_t^F}{\tilde{P}_t^F} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{105}$$

where $\Pi_{t-1}^F = \frac{P_{t-1}^F}{P_{t-2}^F}$.

2.8 Relative prices

The relative price variables move as follows:

$$\tilde{T}_t^N = T_t^N \frac{(1+\alpha^N)^t}{(1+\alpha^T)^t} = \frac{P_t^N}{P_t} \left(\frac{1+\alpha^N}{1+\alpha^T} \right)^t = \frac{\tilde{P}_t^N}{\tilde{P}_t} \tag{106}$$

$$\tilde{T}_t^H = T_t^H = \frac{\tilde{P}_t^H}{\tilde{P}_t} \tag{107}$$

$$\tilde{T}_t^F = T_t^F \left(\frac{1+\alpha^{T^*}}{1+\alpha^T} \right)^t = \frac{\tilde{P}_t^F}{\tilde{P}_t} \tag{108}$$

$$\tilde{T}_t^{N^*} = T_t^{N^*} \left(\frac{1+\alpha^{N^*}}{1+\alpha^{T^*}} \right)^t = \frac{\tilde{P}_t^{N^*}}{\tilde{P}_t^*} \tag{109}$$

$$\tilde{T}_t^{F^*} = T_t^{F^*} = \frac{\tilde{P}_t^F}{\tilde{P}_t^*} \tag{110}$$

$$\tilde{T}_t^{H^*} = T_t^{H^*} \left(\frac{1+\alpha^T}{1+\alpha^{T^*}} \right)^t = \frac{\tilde{P}_t^H}{\tilde{P}_t^*} \tag{111}$$

and the real exchange rate moves as:

$$\widetilde{RER}_t = \frac{\tilde{P}_t^*}{\tilde{P}_t} \tag{112}$$

$$= \frac{P_t^*}{P_t} \left(\frac{1+\alpha^{T^*}}{1+\alpha^T} \right)^t = \frac{\tilde{\mu}_t^*}{\tilde{\mu}_t} \tag{113}$$

$$RER_t = \frac{\tilde{P}_t^*}{\tilde{P}_t} \left(\frac{1+\alpha^T}{1+\alpha^{T^*}} \right)^t \tag{114}$$

2.9 Monetary Policy

In order to abstract from fiscal policy considerations, it is assumed that government spending in the two areas is financed through lump sum taxes. Monetary policy is conducted by the ECB with a Taylor rule that only targets the EMU CPI:

$$R_t = R_{t-1}^{\rho_r} \left(\frac{P_t^{EMU}/P_{t-1}^{EMU}}{\Pi} \right)^{(1-\rho_r)\gamma_\pi} \exp(\varepsilon_t^m). \tag{115}$$

where ε_t^m is an iid monetary policy shock.

2.10 Shocks Processes

$$\log(\tilde{Z}_t^N) = \rho^{Z,N} \log(\tilde{Z}_{t-1}^N) + \varepsilon_t^{Z,N} \tag{116}$$

$$\log(\tilde{Z}_t^T) = \rho^{Z,T} \log(\tilde{Z}_{t-1}^T) + \varepsilon_t^{Z,T} + \varepsilon_t^Z \tag{117}$$

$$\log(\tilde{G}_t^N) = \rho^{G,N} \log(\tilde{G}_t^N) + \varepsilon_t^{G,N} \tag{118}$$

$$\log(\tilde{G}_t^T) = \rho^{G,T} \log(\tilde{G}_t^N) + \varepsilon_t^{G,T} \tag{119}$$

$$\log(\tilde{Z}_t^{N^*}) = \rho^{Z,N^*} \log(\tilde{Z}_{t-1}^{N^*}) + \varepsilon_t^{Z,N^*} \tag{120}$$

$$\log(\tilde{Z}_t^{T^*}) = \rho^{Z,T^*} \log(\tilde{Z}_{t-1}^{T^*}) + \varepsilon_t^{Z,T^*} + \varepsilon_t^{Z} \tag{121}$$

$$\log(\tilde{G}_t^{N^*}) = \rho^{G,N^*} \log(\tilde{G}_t^{N^*}) + \varepsilon_t^{G,N^*} \tag{122}$$

$$\log(\tilde{G}_t^{T^*}) = \rho^{G,T^*} \log(\tilde{G}_t^{T^*}) + \varepsilon_t^{G,T^*} \tag{123}$$

3 Loglinear approximation

Here we present the linearized model that we estimate in the paper. Consumption and production levels, and real wages are normalized by the relevant time trends to make them stationary. Lower case variables denote percent deviations from steady state (i.e. $l_t = \log(L_t) - \log(L) \approx \frac{L_t - L}{L}$), while lower case variables with a tilde denote percent deviations from steady state for those variables normalized by the appropiate level of technology (i.e. $\tilde{c}_t = \log(\tilde{C}_t) - \log(\tilde{C})$, where $\tilde{C}_t = \frac{C_t}{[(1+x)(1+\alpha^T)]^t}$). Also, we set all normalized preference shocks (the $\hat{\xi}' s$ in the previous section) to be a constant. Therefore they do not appear in the loglinearization.

3.1 Euler equation and risk sharing

$$\tilde{\mu}_t = E_t \tilde{\mu}_{t+1} + (r_t - E_t \Delta p_{t+1}) \tag{124}$$

$$-\tilde{\mu}_t \left[1 - \frac{b}{(1+x)(1+\alpha^T)} \right] = \tilde{c}_t - \frac{b}{(1+x)(1+\alpha^T)} \tilde{c}_{t-1} \tag{125}$$

$$-\tilde{\mu}_t^* \left[1 - \frac{b^*}{(1+x)(1+\alpha^{T^*})} \right] = \tilde{c}_t^* - \frac{b^*}{(1+x)(1+\alpha^{T^*})} \tilde{c}_{t-1}^* \tag{126}$$

$$\widetilde{rer}_t = \tilde{\mu}_t^* - \tilde{\mu}_t \tag{127}$$

3.2 Demand functions

Let´s define the following relative prices: $\tilde{t}_t^N = \tilde{p}_t^N - \tilde{p}_t$, $\tilde{t}_t^H = \tilde{p}_t^H - \tilde{p}_t$, $\tilde{t}_t^F = \tilde{p}_t^F - \tilde{p}_t$, $\tilde{t}_t^T = \tilde{p}_t^T - \tilde{p}_t$, $\tilde{t}_t^{N^*} = \tilde{p}_t^{N^*} - \tilde{p}_t^*$, $\tilde{t}_t^{H^*} = \tilde{p}_t^H - \tilde{p}_t^*$, $\tilde{t}_t^{F^*} = \tilde{p}_t^F - \tilde{p}_t^*$, $\tilde{t}_t^{T^*} = \tilde{p}_t^{T^*} - \tilde{p}_t^*$. Then, the consumption demand functions by households are:

$$\tilde{c}_t^H = -\nu\tilde{t}_t^H - (\varepsilon - \nu)\tilde{t}_t^T + \tilde{c}_t \tag{128}$$

$$\tilde{c}_t^F = -\nu\tilde{t}_t^F - (\varepsilon - \nu)\tilde{t}_t^T + \tilde{c}_t \tag{129}$$

$$\tilde{c}_t^N = -\varepsilon\tilde{t}_t^N + \tilde{c}_t \tag{130}$$

$$\tilde{c}_t^{H^*} = -\nu\tilde{t}_t^{H^*} - (\varepsilon - \nu)\tilde{t}_t^{T^*} + \tilde{c}_t^* \tag{131}$$

$$\tilde{c}_t^{F^*} = -\nu\tilde{t}_t^{F^*} - (\varepsilon - \nu)\tilde{t}_t^{T^*} + \tilde{c}_t^* \tag{132}$$

$$\tilde{c}_t^{N^*} = -\varepsilon\tilde{t}_t^{N^*} + \tilde{c}_t^* \tag{133}$$

3.3 Labor supply

$$\varpi l_t = \tilde{w}_t + \tilde{\mu}_t \tag{134}$$

$$\varpi^* l_t^* = \tilde{w}_t^* + \tilde{\mu}_t^* \tag{135}$$

The hours allocation across sectors is:

$$l_t = (1-\gamma) l_t^N + \gamma l_t^T \tag{136}$$

$$l_t^* = (1-\gamma^*) l_t^{N^*} + \gamma^* l_t^{T^*} \tag{137}$$

3.4 Technology

The production functions in the two sectors are:

$$\tilde{y}_t^N = \tilde{z}_t^N + l_t^N, \tag{138}$$

and

$$\tilde{y}_t^H = \tilde{z}_t^T + l_t^T. \tag{139}$$

In the foreign country:

$$\tilde{y}_t^{N^*} = \tilde{z}_t^{N^*} + l_t^{N^*}, \tag{140}$$

and

$$\tilde{y}_t^{F^*} = \tilde{z}_t^{T^*} + l_t^{T^*}. \tag{141}$$

3.5 Price setting

In the four sectors:

$$\Delta\tilde{p}_t^N - \varphi_N \Delta\tilde{p}_{t-1}^N = \frac{(1-\theta_N)\left[1-\beta\theta_N\left(\frac{1+\alpha_N}{1+\alpha_T}\right)\right]}{\theta_N}(\tilde{w}_t - \tilde{t}_t^N - \tilde{z}_t^N) + \beta\left(\frac{1+\alpha_N}{1+\alpha_T}\right)\left(\Delta\tilde{p}_{t+1}^N - \varphi_N \Delta\tilde{p}_t^N\right) \quad (142)$$

$$\Delta\tilde{p}_t^H - \varphi_H \Delta\tilde{p}_{t-1}^H = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}(\tilde{w}_t - \tilde{t}_t^H - \tilde{z}_t^T) + \beta\left(\Delta\tilde{p}_{t+1}^H - \varphi_H \Delta\tilde{p}_t^H\right) \quad (143)$$

$$\Delta\tilde{p}_t^{N^*} - \varphi_{N^*} \Delta\tilde{p}_{t-1}^{N^*} = \frac{(1-\theta_{N^*})\left[1-\beta\theta_{N^*}\left(\frac{1+\alpha_{N^*}}{1+\alpha_{T^*}}\right)\right]}{\theta_{N^*}}(\tilde{w}_t^* - \tilde{t}_t^{N^*} - \tilde{z}_t^{N^*}) + \beta\left(\frac{1+\alpha_{N^*}}{1+\alpha_{T^*}}\right)\left(\Delta\tilde{p}_{t+1}^{N^*} - \varphi_{N^*} \Delta\tilde{p}_t^{N^*}\right) \quad (144)$$

$$\Delta\tilde{p}_t^F - \varphi_{F^*} \Delta\tilde{p}_{t-1}^F = \frac{(1-\theta_{F^*})(1-\beta\theta_{F^*})}{\theta_{F^*}}(\tilde{w}_t^* - \tilde{t}_t^{F^*} - \tilde{z}_t^{T^*}) + \beta\left(\Delta\tilde{p}_{t+1}^F - \varphi_{F^*} \Delta\tilde{p}_t^F\right) \quad (145)$$

3.6 Relevant price indices

The Spanish consumer price inflation is given by

$$\Delta\tilde{p}_t = \gamma\lambda\Delta\tilde{p}_t^H + \gamma(1-\lambda)\Delta\tilde{p}_t^F + (1-\gamma)\Delta\tilde{p}_t^N \quad (146)$$

The rest of euro area aggregates are given by:

$$\Delta\tilde{p}_t^* = \gamma^*(1-\lambda^*)\Delta\tilde{p}_t^H + \gamma^*\lambda^*\Delta\tilde{p}_t^F + (1-\gamma^*)\Delta\tilde{p}_t^{N^*} \quad (147)$$

The European Central Bank targets the euro-area wide CPI, which is assumed to be an harmonic mean of the price levels of Spain and the rest of the euro area:

$$\Delta\tilde{p}_t^{EMU} = s\Delta\tilde{p}_t + (1-s)\Delta\tilde{p}_t^* \quad (148)$$

using the country sizes as weights.

The relative price variables move as follows:

$$\tilde{t}_t^N = \tilde{t}_{t-1}^N + \Delta\tilde{p}_t^N - \Delta\tilde{p}_t \tag{149}$$

$$\tilde{t}_t^T = \tilde{t}_{t-1}^T + \Delta\tilde{p}_t^T - \Delta\tilde{p}_t \tag{150}$$

$$\tilde{t}_t^H = \tilde{t}_{t-1}^H + \Delta\tilde{p}_t^H - \Delta\tilde{p}_t \tag{151}$$

$$\tilde{t}_t^F = \tilde{t}_{t-1}^F + \Delta\tilde{p}_t^F - \Delta\tilde{p}_t \tag{152}$$

$$\tilde{t}_t^{N^*} = \tilde{t}_{t-1}^{N^*} + \Delta\tilde{p}_t^{N^*} - \Delta\tilde{p}_t^* \tag{153}$$

$$\tilde{t}_t^{T^*} = \tilde{t}_{t-1}^{T^*} + \Delta\tilde{p}_t^{T^*} - \Delta\tilde{p}_t^* \tag{154}$$

$$\tilde{t}_t^{F^*} = \tilde{t}_{t-1}^{F^*} + \Delta\tilde{p}_t^F - \Delta\tilde{p}_t^* \tag{155}$$

$$\tilde{t}_t^{H^*} = \tilde{t}_{t-1}^{H^*} + \Delta\tilde{p}_t^H - \Delta\tilde{p}_t^* \tag{156}$$

and the real exchange rate moves as:

$$\widetilde{rer}_t = \widetilde{rer}_{t-1} + \Delta\tilde{p}_t^* - \Delta\tilde{p}_t \tag{157}$$

3.7 Market Clearing

The market clearing for the nontradable goods sector at home is

$$\tilde{y}_t^N = (1-\eta)\tilde{c}_t^N + \eta\tilde{g}_t^N \tag{158}$$

where $\eta = \frac{G^N}{Y^N} = \frac{G^T}{Y^H}$ is the fraction of government spending activities over output in the steady state.

The market clearing condition in the tradable goods sector at home is

$$\tilde{y}_t^H = (1-\eta)[\lambda\tilde{c}_t^H + (1-\lambda)\tilde{c}_t^{H^*}] + \eta\tilde{g}_t^T \tag{159}$$

For the foreign country, these expressions become:

$$\tilde{y}_t^{N^*} = (1-\eta^*)c_t^{N^*} + \eta^* g_t^{N^*} \tag{160}$$

and

$$\tilde{y}_t^{F^*} = (1-\eta^*)[(1-\lambda^*)\tilde{c}_t^F + \lambda^*\tilde{c}_t^{F^*}] + \eta^*\tilde{g}_t^{T^*} \tag{161}$$

Aggregate real GDP aggregates tradable and nontradable goods using the appropiate relative prices:

$$\tilde{y}_t = \gamma(\tilde{t}_t^H + \tilde{y}_t^H) + (1-\gamma)(\tilde{t}_t^N + \tilde{y}_t^N) \tag{162}$$

$$\tilde{y}_t^* = \gamma^*(\tilde{t}_t^{F^*} + y_t^{F^*}) + (1-\gamma^*)(\tilde{t}_t^{N^*} + y_t^{N^*}) \tag{163}$$

3.8 Monetary Policy

In order to abstract from fiscal policy considerations, it is assumed that government spending in the two areas is financed through lump sum taxes. Monetary policy is conducted by the ECB with a Taylor rule that only targets the EMU CPI:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\gamma_\pi \Delta\tilde{p}_t^{EMU} + \varepsilon_t^m \tag{164}$$

3.9 Shocks

The technology shocks evolve as:

$$\tilde{z}_t^T = \rho_{t-1}^{Z,T}\tilde{z}_{t-1}^T + \varepsilon_t^{Z,T} + \varepsilon_t^Z \tag{165}$$

$$\tilde{z}_t^N = \rho^{Z,N}\tilde{z}_{t-1}^N + \varepsilon_t^{Z,N} \tag{166}$$

$$\tilde{z}_t^{T^*} = \rho^{Z,T^*}\tilde{z}_{t-1}^{T^*} + \varepsilon_t^{Z,T^*} + \varepsilon_t^Z \tag{167}$$

$$\tilde{z}_t^{N^*} = \rho^{Z,N^*}\tilde{z}_{t-1}^{N^*} + \varepsilon_t^{Z,N^*} \tag{168}$$

The demand shocks are:

$$\tilde{g}_t^T = \rho^{G,T}\tilde{g}_{t-1}^T + \varepsilon_t^{G,T} \tag{169}$$

$$\tilde{g}_t^N = \rho^{G,N}\tilde{g}_{t-1}^N + \varepsilon_t^{G,N} \tag{170}$$

$$\tilde{g}_t^{T*} = \rho^{G,T*}\tilde{g}_{t-1}^{T*} + \varepsilon_t^{G,T*} \tag{171}$$

$$\tilde{g}_t^{N*} = \rho^{G,N*}\tilde{g}_{t-1}^{N*} + \varepsilon_t^{G,N*} \tag{172}$$

Finally, the innovation to the Taylor rule is ε_t^m.

3.10 Relationship between variables in the model and in the data

In order to be able to relate the observed variables in the data model with those in the model, first we need to note that:

$$\begin{aligned} P_t(1+\alpha^T)^t &= P_t^N(1+\alpha^N)^t = P_t^H(1+\alpha^T)^t = P_t^F(1+\alpha^{T^*})^t \\ P_t^*(1+\alpha^{T^*})^t &= P_t^{N^*}(1+\alpha^{N^*})^t = P_t^H(1+\alpha^T)^t = P_t^F(1+\alpha^{T^*})^t \end{aligned}$$

Note that the α's give the differential inflation rate with respect to the EMU aggregate,

Therefore, the variable $\tilde{P}_t^N = P_t^N\frac{(1+\alpha^N)^t}{\Pi^t}$ does not grow.Then, we have the following relationship in logs:

$$\tilde{p}_t^N + t\left(\alpha^N - \pi\right) = p_t^N$$

where $\pi = \log(\Pi)$. We denote by the superscript *obs* the variable as it is defined in the data:

$$\begin{aligned} \Delta p_t^{N,OBS} &= \pi - \alpha^N + \Delta\tilde{p}_t^N \\ \Delta p_t^{N^*,OBS} &= \pi - \alpha^{N^*} + \Delta\tilde{p}_t^{N^*} \\ \Delta p_t^{OBS} &= \pi - \alpha^T + \Delta\tilde{p}_t \\ \Delta p_t^{*,OBS} &= \pi - \alpha^{T^*} + \Delta\tilde{p}_t^* \end{aligned}$$

where $\pi = \log(\Pi)$. For output growth:

$$\begin{aligned}
\Delta y_t^{N,OBS} &= x + \alpha^N + \Delta \tilde{y}_t^N \\
\Delta y_t^{OBS} &= x + \alpha^T + \Delta \tilde{y}_t \\
\Delta y_t^{N^*,OBS} &= x + \alpha^{N^*} + \Delta \tilde{y}_t^{N^*} \\
\Delta y_t^{*,OBS} &= x + \alpha^{T^*} + \Delta \tilde{y}_t^*
\end{aligned}$$

4 Estimating the model with levels of variables[1]

In this section I present the results from estimating the model when all price and quantity variables are introduced in levels in the measurement equations, which now look as follows:

$$\begin{aligned}
p_t^{N,OBS} &= p_{t-1}^{N,OBS} + \pi - \alpha^N + \Delta \tilde{p}_t^N \\
p_t^{N^*,OBS} &= p_{t-1}^{N^*,OBS} + \pi - \alpha^{N^*} + \Delta \tilde{p}_t^{N^*} \\
p_t^{OBS} &= p_{t-1}^{OBS} + \pi - \alpha^T + \Delta \tilde{p}_t \\
p_t^{*,OBS} &= p_{t-1}^{*,OBS} + \pi - \alpha^{T^*} + \Delta \tilde{p}_t^*
\end{aligned}$$

where $\pi = \log(\Pi)$. For output growth:

$$\begin{aligned}
y_t^{N,OBS} &= y_{t-1}^{N,OBS} + x + \alpha^N + \Delta \tilde{y}_t^N \\
y_t^{OBS} &= y_{t-1}^{OBS} + x + \alpha^T + \Delta \tilde{y}_t \\
y_t^{N^*,OBS} &= y_{t-1}^{N^*,OBS} + x + \alpha^{N^*} + \Delta \tilde{y}_t^{N^*} \\
y_t^{*,OBS} &= y_{t-1}^{*,OBS} + x + \alpha^{T^*} + \Delta \tilde{y}_t^*
\end{aligned}$$

Since the measurement equations includes nonstationary variables, a diffuse prior is needed to initialize the Kalman filter. DYNARE uses the Koopans and Durbin (2001) algorithm.

The estimated parameter values are the following for the two models. Note that they are not so different from those reported in Table 5 in the main text. Since we calibrate most parameter values related to the size of the sectors, and the model

[1] I am grateful to Michel Juillard for help in DYNARE programming.

does not have capital, it appears that estimating the model in levels does not add additional information with respect to the estimation with variables in first differences.

Table A.1. Posterior Distributions, Variables in Levels

	Restricted	*Unrestricted*		*Restricted*	*Unrestricted*
θ_H	0.38 (0.25- 0.52)	0.34 (0.21- 0.44)	γ_Π	1.45 (1.27 - 1.66)	1.44 (1.25 - 1.64)
θ_{F^*}	0.46 (0.33- 0.59)	0.42 (0.29- 0.55)	ρ_R	0.73 (0.67 - 0.78)	0.69 (0.64 - 0.74)
θ_N	0.66 (0.57 - 0.77)	0.66 (0.59 - 0.75)	$\rho^{Z,T}$	0.67 (0.54- 0.82)	0.70 (0.56- 0.87)
θ_{N^*}	0.88 (0.82 - 0.96)	0.86 (0.83 - 0.90)	$\rho^{G,T}$	0.96 (0.94- 0.98)	0.86 (0.82- 0.90)
φ_H	0.34 (0.04 - 0.62)	0.46 (0.13 - 0.81)	$\rho^{Z,N}$	0.78 (0.66 - 0.89)	0.78 (0.67 - 0.91)
φ_{F^*}	0.38 (0.07 - 0.69)	0.36 (0.06 - 0.66)	$\rho^{G,N}$	0.98 (0.97 - 0.99)	0.89 (0.85 - 0.93)
φ_N	0.40 (0.08 - 0.72)	0.24 (0.04 - 0.45)	$\sigma(\varepsilon_t^m)$	0.14 (0.11 - 0.18)	0.13 (0.09 - 0.16)
φ_{N^*}	0.46 (0.1 - 0.7)	0.44 (0.17 - 0.70)	$\sigma(\varepsilon_t^Z)$	0.52 (0.34 - 0.70)	0.52 (0.35 - 0.69)
ε	0.49 (0.21 - 0.83)	0.51 (0.25 - 0.77)	$\sigma(\varepsilon_t^{Z,T})$	0.60 (0.40 - 0.80)	0.57 (0.37 - 0.77)
ν	0.52 (0.14 - 0.92)	1.23 (0.48 - 1.95)	$\sigma(\varepsilon_t^{Z,T^*})$	0.57 (0.34 - 0.82)	0.45 (0.26 - 0.63)
x	0.55 (0.47 - 0.63)	–	$\sigma(\varepsilon_t^{Z,N})$	0.75 (0.53 - 0.98)	0.71 (0.48 - 0.91)
π	0.50 (0.46 - 0.54)	–	$\sigma(\varepsilon_t^{Z,N^*})$	0.89 (0.60 - 1.22)	0.88 (0.61 - 1.15)
α_N	-0.13 (-0.19 - 0.08)	–	$\sigma(\varepsilon_t^{G,T})$	3.10 (2.55 - 3.65)	2.31 (1.91 - 2.67)
α_T	0.04 (0 - 0.08)	–	$\sigma(\varepsilon_t^{G,T^*})$	3.17 (2.61 - 3.72)	2.61 (2.16 - 3.03)
α_{N^*}	−0.06 (-0.10 - -0.02)	–	$\sigma(\varepsilon_t^{G,N})$	4.56 (3.83 - 5.27)	2.69 (2.28 - 3.15)
α_{T^*}	0.07 (0.02 - 0.11)	–	$\sigma(\varepsilon_t^{G,N^*})$	2.33 (1.89 - 2.78)	1.93 (1.61 - 2.25)
Log-L	-104.84	-24.63			

5 Additional second moments in the data and in the models

In the following tables, we present contemporaneous correlations, and autocorrelations of each variable, in the data, and their posterior mean counterparts in the two estimated models. The models' implications are not so different regarding the fit of contemporaneous correlations and autocorrelations. Both models do a good job in fitting the correlations between each of the inflation indicators, and also do a good job in capturing the near-zero correlations between most inflation indicators and real growth indicators (Table A.2). None of the models does a good job in capturing the large and positive correlation of real growth variables: the models predict positive correlations, but not as large as in the data. Also the models do not do a very good job at capturing the contemporaneous correlation between the nominal interest rate and the rest of EMU (inflation and growth) variables.

Table A.2. Correlations in the data and in the models

	Δp_t	Δp_t^N	Δp_t^*	$\Delta p_t^{N,*}$	Δy_t
Δp_t^N					
Data	0.38	1			
Restricted	0.68	1			
Unrestricted	0.62	1			
Δp_t^*					
Data	0.56	0.30	1		
Restricted	0.49	0.27	1		
Unrestricted	0.50	0.27	1		
$\Delta p_t^{N,*}$					
Data	0.01	0.29	0.24	1	
Restricted	0.21	0.30	0.52	1	
Unrestricted	0.22	0.31	0.27	1	
Δy_t					
Data	-0.01	0.17	-0.09	-0.19	1
Restricted	0.06	0.13	-0.16	-0.03	1
Unrestricted	-0.16	-0.01	-0.18	-0.02	1

Table A.2 (cont.). Correlations in the data and in the models

	Δp_t	Δp_t^N	Δp_t^*	$\Delta p_t^{N,*}$	Δy_t	Δy_t^N	Δy_t^*	$\Delta y_t^{N,*}$
Δy_t^N								
Data	0.16	0.10	0.02	-0.14	0.74	1		
Restricted	0.15	0.07	-0.01	-0.01	0.68	1		
Unrestricted	0.06	-0.04	-0.03	-0.01	0.50	1		
Δy_t^*								
Data	-0.01	0.12	0.06	-0.12	0.54	0.14	1	
Restricted	-0.13	-0.04	-0.07	-0.04	0.18	0.08	1	
Unrestricted	-0.07	0.03	-0.09	-0.01	0.22	0.09	1	
$\Delta y_t^{N,*}$								
Data	0.11	0.12	0.02	-0.15	0.51	0.21	0.87	1
Restricted	0.06	0.05	0.04	-0.08	0.15	0.22	0.41	1
Unrestricted	0.03	0.03	0.03	-0.09	0.13	0.18	0.46	1
r_t								
Data	-0.11	0.05	0.03	0.11	0.17	-0.11	0.42	0.47
Restricted	0.10	0.06	0.29	0.35	-0.18	-0.08	-0.25	-0.21
Unrestricted	0.19	0.13	0.39	0.44	-0.21	-0.12	-0.25	-0.19

When it comes to matching persistence of the observed variables, both models deliver higher inflation persistence and lower output growth persistence than in the data (Table A.3). They both do a particularly good job in matching the persistence of nontradable inflation in the rest of the EMU, and the nominal interest rates.

Table A.3: Correlogram in the data and in the models

	1	2	3	4	5
Δp_t					
Data	0.07	0.05	0.03	-0.02	0.01
Restricted	0.34	-0.02	-0.09	-0.07	-0.03
Unrestricted	0.35	-0.02	-0.1	-0.07	-0.06
Δp_t^N					
Data	0.20	-0.05	-0.16	-0.15	-0.02
Restricted	0.62	0.31	0.12	0.03	-0.01
Unrestricted	0.63	0.32	0.13	0.03	-0.02
Δp_t^*					
Data	0.1	0.09	0.42	-0.08	0.03
Restricted	0.40	0.07	-0.01	-0.01	0.01
Unrestricted	0.39	0.06	-0.01	-0.01	0.01
$\Delta p_t^{N,*}$					
Data	0.87	0.75	0.56	0.30	0.13
Restricted	0.86	0.67	0.50	0.35	0.24
Unrestricted	0.86	0.68	0.50	0.36	0.25

Table A.3 (cont.): Correlogram in the data and in the models

	1	2	3	4	5
Δy_t					
Data	0.36	0.28	0.34	0.05	0.20
Restricted	0.02	-0.05	-0.06	-0.05	-0.04
Unrestricted	0.05	-0.04	-0.08	-0.08	-0.06
Δy_t^N					
Data	0.13	0.27	0.30	-0.05	0.24
Restricted	-0.01	-0.03	-0.04	-0.05	-0.06
Unrestricted	-0.01	-0.04	-0.05	-0.05	-0.4
Δy_t^*					
Data	0.43	0.43	0.12	0.07	-0.02
Restricted	0.00	-0.05	-0.06	-0.05	-0.04
Unrestricted	0.00	-0.05	-0.07	-0.06	-0.05
$\Delta y_t^{N,*}$					
Data	0.25	0.41	0.16	0.32	0.02
Restricted	0.03	0.02	-0.01	-0.04	-0.05
Unrestricted	-0.01	-0.05	-0.07	-0.06	-0.05
r_t					
Data	0.89	0.75	0.60	0.45	0.32
Restricted	0.74	0.52	0.38	0.29	0.25
Unrestricted	0.76	0.53	0.37	0.27	0.21