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Cointegrated TFP processes and international business cycles

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ABSTRACT

A puzzle in international macroeconomics is that real exchange rates are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact. This paper provides evidence that TFP processes for the U.S. and the "rest of the world" are characterized by a vector error correction model (VECM) and that adding cointegrated technology shocks to the standard IRBC model helps to explain the observed high real exchange rate volatility. Also, the model can explain the observed increase in real exchange rate volatility with respect to output in the last 20 years by changes in the parameters of the VECM.

1. Introduction

A central puzzle in international macroeconomics is that observed real exchange rates (RERs) are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations. For instance, Heathcote and Perri (2002) simulate a two-country, two-good economy with the total factor productivity (TFP) shocks and find that the model can explain less than a fourth of the observed relative volatility of the RER with respect to output for United States (U.S.) data. An important feature of their model, following the seminal work of Backus et al. (1992) and Baxter and Crucini (1993), is that it considers stationary TFP shocks that follow a vector autoregression (VAR) process in levels.[1,2]

This paper provides evidence that TFP processes for the U.S. and the "rest of the world" (R.W.) have a unit root and are cointegrated. Motivated by this empirical finding, technology shocks that follow a vector error correction model (VECM) process are introduced into an otherwise standard two-country, two-good model. Engle and Granger (1987) indicate that if the system under study includes integrated variables and cointegrating relationships, then this system will be more appropriately specified as a VECM rather than a VAR in levels. As Engle and Granger (1987) note, estimating a VAR in levels for cointegrated systems ignores important constraints on the coefficient matrices. Although these constraints are satisfied asymptotically, small sample improvements are likely to result from imposing them on the cointegrating relationships.

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[1] We provide an online appendix with the full set of normalized equilibrium conditions and the necessary replication files.

[2] Other studies that consider a VAR in levels are Kehoe and Perri (2002), Dotsey and Duarte (2009), Corsetti et al. (2008a,b), and Heathcote and Perri (2009).

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The presence of cointegrated TFP shocks requires restrictions on preferences, production functions, and the law of motion of the shocks for the balanced growth path to be chosen by optimizing agents. The restrictions on preferences and technology of King et al. (1988) are sufficient for the existence of balanced growth in a closed economy. However, in a two-country model, an additional restriction on the cointegrating vector related to the TFP processes is needed. In particular, the cointegrating vector must be $(1,-1)$, which means the ratio of TFP processes (or, equivalently, the log difference of TFP processes) across countries is stationary. After presenting evidence supporting this additional restriction, the simulated model with a VECM specification for TFP processes solves a large part of the RER volatility puzzle without affecting the good match for other moments of domestic and international variables. In particular, the model can generate a relative volatility of the RER more than two times larger than an equivalent model with stationary shocks calibrated as in Heathcote and Perri (2002).

Why does a model with cointegrated TFP shocks generate higher relative volatility of the RER than a model with stationary shocks? The reason is that the VECM parameter estimates imply higher persistence and lower spillovers than the traditional stationary calibrations, which translates into higher persistence of the TFP differential across countries. This is the crucial feature of the model that helps explain the relative volatility of the RER with respect to output. The mechanism works as follows. When a positive TFP shock hits the home economy, output, consumption, investment and hours work increase in the home economy, and the RER depreciates as home prices decrease. In the foreign country, output, investment and hours decrease while consumption increases, as foreign households anticipate the arrival of the technology improvement in the future and face a wealth effect. What happens as the persistence of the TFP differential increases, or, put differently, the speed of transmission of the shocks decreases? It delays the arrival of the TFP improvement in the foreign country and it increases the persistence of the TFP shock in the home country. This delay attenuates the wealth effect for the foreign economy and strengthens it for the home country. As a result, labor, investment and output respond less strongly, reducing output volatility. The increased persistence in TFP differential also reduces the supply of the home good and increases the demand for foreign intermediate goods (which are produced at a higher cost, because the foreign country has not received the productivity shock yet), which leads to an even larger RER depreciation, increasing RER volatility.

Another very well-documented empirical fact is the substantial decline in the volatility of most U.S. macroeconomic variables during the last 20 years, a phenomenon known as the “Great Moderation”.[3] The Great Moderation has not affected the RER as strongly as it has affected output. As a result, the ratio of RER volatility to output volatility has increased. The increase in the relative volatility of the RER of the U.S. dollar coincides in time with a weakening of the cointegrating relationship of TFP shocks between the U.S. and the R.W.[4] More importantly, this paper confirms that by allowing the cointegrating relationship to change as it does in the data, the model can jointly account for the observed increase in the relative volatility of the RER and the substantial decline in the volatility of output.

Baxter and Stockman (1989) showed that changes in the nominal exchange rate regime greatly affected volatility of the RER but had almost no effect on output volatility and other macroeconomic variables. This empirical fact is an important challenge for IRBC models, since they assume a tight relationship between RER and output volatilities. Since the model is based on the standard IRBC framework, it suffers from the same limitation. To minimize the effect of this valid criticism, and since the impact of changes in the nominal exchange rate regime cannot be studied, the focus of the analysis is the post Bretton-Woods period of flexible nominal exchange rates.

Our paper relates to two important strands of the literature. On the one hand, it connects with the literature stressing the importance of stochastic trends to explain economic fluctuations. King et al. (1991) find that a common stochastic trend explains the co-movements of the main U.S. real macroeconomic variables. Lastrapes (1992) reports that fluctuations in real and nominal exchanges rates are primarily due to permanent real shocks. Engel and West (2005) show that RERs manifest near-random-walk behavior if TFP processes are random walks and the discount factor is near one, while Nason and Rogers (2008) generalize this hypothesis to a larger class of models. Aguiar and Gopinath (2007) show that trend shocks are the primary source of fluctuations in emerging economies. Alvarez and Jermann (2005) and Corsetti et al. (2008a) highlight the importance of persistent disturbances to explain asset prices and RER fluctuations, respectively. Also, Lubik and Schorfheide (2005) and Rabanal and Tuesta (2010) introduce random walk TFP shocks to explain international fluctuations, and Justiniano and Preston (2010) suggest that it is important to introduce correlations between the innovations of several structural shocks in order to explain the co-movement between Canadian and U.S. macroeconomic variables. However, these papers do not formalize a VECM, test for cointegration, or estimate the cointegrating vector.

On the other hand, this paper also links to the literature analyzing different mechanisms to understand RER fluctuations. Some recent papers study the effects of monetary shocks and nominal rigidities. Chari et al. (2002) are able to explain RER volatility in a monetary model with sticky prices and a high degree of risk aversion. However, their model achieves success by increasing the variance of monetary shocks beyond what the data indicate. Benigno (2004) focuses on the role of interest rate inertia and asymmetric nominal rigidities across countries. Other papers use either non-traded goods, pricing to market, or some form of distribution costs (see Corsetti et al., 2008a,b; Benigno and Thoenissen, 2008; Dotsey and Duarte, 2009). Our model includes only tradable goods with home bias, which is the only

[3] Some early discussion of the Great Moderation can be found in Kim and Nelson (1999). A discussion of different interpretations for this phenomenon and some international evidence can be found in Stock and Watson (2003) and Stock and Watson (2005), respectively.

[4] In Section 4, we describe the set of countries that compose our definition of R.W.

source of RER fluctuations. Our choice is guided by evidence that the relative price of tradable goods has large and persistent fluctuations that explain most of the RER volatility (see Engel, 1993, 1999). Fluctuations of the relative price of non-traded goods accounts for, at most, one-third of RER volatility (see Betts and Kehoe, 2006; Burstein et al., 2006; Rabanal and Tuesta, 2007). In any case, this choice causes an empirical problem. Our measure of RER is based on the consumer price indices (CPIs) that include non-traded goods, while the model does not have a non-traded goods sector. To reduce the gap between model and data, two alternative measures of RER are considered. The first measure is constructed using producer price indices (PPIs) and the second from export deflators. Of course, the two series still maintain some gap between theory and measurement, but the role of non-traded goods is reduced. Using these two other measures, the results do not change.

The rest of the paper is organized as follows. Section 2 documents the increase in the RER volatility with respect to output volatility for the U.S. Section 3 presents the model with cointegrated TFP shocks. Section 4 reports estimates for the law of motion of TFP processes for the U.S. and the R.W. Section 5 discusses the main findings from simulating the model, leaving Section 6 for concluding remarks.

2. RER volatility and the Great Moderation

This section presents evidence that, in the period known as ''the Great Moderation'', the relative volatility of the RER (measured as the real effective exchange rate) with respect to output (measured as real GDP) has increased in the U.S.[5] The real effective exchange rate is constructed as a geometric average of bilateral CPI-based RERs with respect to the Euro area, Japan, Canada, the United Kingdom and Australia, with the same weights used by the Federal Reserve to construct its real effective exchange rate of the U.S. dollar series. These countries represented 69 percent of the aggregate weight in 1973 and 46 percent in 2009. These countries were chosen to be consistent with the definition of the R.W. later in the paper. The RER series is thus constructed from 1957:1 to 2010:1, proceeding in two steps. Between 1973:4 and 2010:1, series for nominal exchange rates with the U.S. dollar were obtained from the Federal Reserve and each country's CPI series was extracted through the IMF's International Financial Statistics (IFS). The only exception was the Euro area, where the source for the CPI is the area wide model (AWM) of the European Central Bank (ECB).

The Federal Reserve does not publish bilateral exchange rate data prior to 1973. In addition, the Federal Reserve weights start in 1973. Moreover, the AWM from which the Euro area CPI is obtained starts in 1970. The real effective exchange rate of the U.S. dollar between 1957:1 and 1973:3 is extended backwards as follows. A Euro area RER aggregate using U.S. dollar nominal exchange rates and CPI data for West Germany, France, Italy, Spain and the Netherlands from the IMF's IFS was constructed, using the Federal Reserve's weights in 1973 (the first year these weights are available). These five countries represented about 86 percent of Euro area trade with the U.S. in 1973. Then, the Euro area RER was averaged with the bilateral RERs of Japan, Canada, the United Kingdom and Australia using again 1973 weights. Since both series overlap in 1973:4, this date was chosen to normalize and put both series together.

Fig. 1 shows the standard deviation of the HP-filtered output series, the standard deviation of the HP-filtered RER series, and the ratio of the two.[6] Standard deviations are computed using rolling windows of 40 quarters and data from 1957:1 to 2010:1, so Fig. 1 displays volatilities between 1966:4 and 2010:1. The figure shows a substantial decline in the volatility of output from around 2 percent standard deviation until the mid 1990s to 1 percent after that date. This decline in output volatility is what is typically referred to as ''the Great Moderation''. The volatility of the RER follows a different path: the standard deviation was at about 4 percent until the mid 1980s, increased to around 7 percent for the 1990s, and again declined to around 4 percent after that.

What is the behavior of the ratio of volatilities between the RER and output? The ratio has increased in a non-monotonic way from around 1 percent to around 4 percent in the period of study. Hence, the volatility of the RER has risen by a factor of four relative to that of output. But this is not a formal test of a structural break. To perform such a test, the ratio of RER to output volatility is modeled as an autoregressive (AR) process of order one with mean. The Quandt–Andrews unknown breakpoint test is used on the estimated mean parameter (trimming 15 percent of the data), using the sample 1966:4 to 2010:1. The results are reported in Table 1 and clearly reject the null hypothesis of no breakpoints in the data: the exponential likelihood ratio is well above the 5 percent critical value. In addition, using the maximum likelihood ratio *F*-statistic, the date with the highest probability of a break is 1993:4.

The rest of the paper builds a two-country, two-good model that is calibrated using standard parameters of the IRBC literature and estimated parameters of a VECM using TFP processes for the U.S. and the R.W. After seeing the evidence presented in Fig. 1, the reader may anticipate that the goal of the paper is to match time-varying targets. That it is not the case. Section 5.4 below shows that by estimating two separate VECMs with a breakpoint in 1993:4, the resulting parameter estimates explain a large fraction of the observed increase in the relative volatility of the RER with respect to output.

[5] Similar behavior can be observed for the United Kingdom, Canada, and Australia. We do not present those graphs because of space considerations.

[6] Using the Federal Reserve's index for the real effective exchange rate of the U.S. Dollar delivers similar results. The Fed's measure of the RER of the U.S. Dollar is available from January 1973. The correlation between the HP-filtered RER constructed by the Fed and the measure based in the five countries is 0.94.

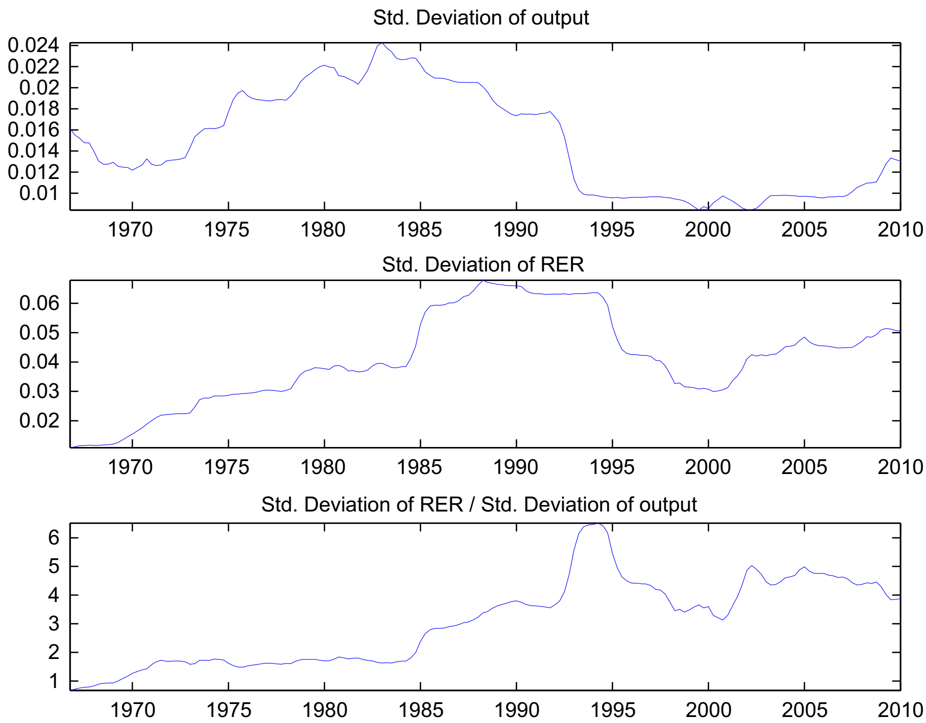

Fig. 1. Standard deviation of HP-filtered output and RER and the ratio of standard deviations.

Table 1
Quandt–Andrews test.

Stability of relative volatility of the RER		
Method	*F*-statistic	Prob.
Maximum likelihood ratio (1993:4)	15.36773	0.0021
Exponential likelihood ratio	4.344542	0.0018

Notes: The null hypothesis is "No breakpoints within trimmed data". Equation sample: 1967:1 2010:1. Test sample: 1973:3 2003:3 (given 15 percent trimming). Number of breaks compared: 121.

3. The model

This section presents a standard two-country, two-good IRBC model similar to that in Heathcote and Perri (2002). The main difference with respect to the standard IRBC literature is the definition of the stochastic processes for the log of TFP.[7] In that literature, the TFP processes of the two countries are assumed to be stationary or trend stationary in logs, and are modeled as a VAR process. Baxter and Crucini (1995) were the first paper to consider permanent shocks and the possibility of cointegration in the context of this class of models, but they did not pursue the VECM specification because the evidence of cointegration was mixed for the bilateral pairs they studied. In the present model, TFP processes that are cointegrated of order C(1,1) are introduced, which implies that the processes are integrated of order one but a linear combination is stationary. According to the Granger representation theorem, the C(1,1) assumption is equivalent to defining a VECM for the law of motion of the differences of the TFP processes.[8] The VECM is defined in more detail in Section 3.2.3. The cointegration assumption has strong and testable implications for the data. The empirical evidence supporting this assumption will be presented in Section 4.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use domestic capital and labor in the production process. The final good can only be used for consumption or investment in the domestic economy. The stock of domestic capital can therefore only be increased by combining domestic and foreign goods. Thus, all

[7] To avoid bothersome repetition, in the remainder of the paper the concept "TFP" will actually mean "the log of TFP".
[8] See Engle and Granger (1987).

trade between countries occur at the intermediate goods level. In addition, consumers trade non-contingent international riskless bonds denominated in units of domestic intermediate goods. No other financial asset is available. In each period t, the economy experiences one of finitely many events s_t, with history $s^t=(s_0,\dots,s_t)$ of events up through period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$ and s_0 is given. We only present the problem of the home country. The problem faced by foreign country households and firms is symmetric, and hence it is not presented.

3.1. Households

The representative household of the home country solves

$$\max_{\{C(s^t),L(s^t),X(s^t),K(s^t),D(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \frac{\{C(s^t)^{\tau}[1-L(s^t)]^{1-\tau}\}^{1-\sigma}}{1-\sigma}, \tag{1}$$

subject to the following budget constraint

$$C(s^t)+X(s^t)+\frac{P_H(s^t)}{P(s^t)}\overline{Q}(s^t)D(s^t) \le W(s^t)L(s^t)+R(s^t)K(s^{t-1})+\frac{P_H(s^t)}{P(s^t)}\{D(s^{t-1})-\Phi[D(s^t)]\}, \tag{2}$$

and the law of motion for capital

$$K(s^t)=(1-\delta)K(s^{t-1})+X(s^t). \tag{3}$$

The following notation is used: $\beta \in (0,1)$ is the discount factor, $L(s^t) \in (0,1)$ is the fraction of time allocated to work in the home country, $C(s^t) \ge 0$ are units of consumption of the final good, $X(s^t) \ge 0$ are units of investment, and $K(s^t) \ge 0$ is the capital stock in the home country at the beginning of period $t+1$. $P(s^t)$ is the price of the home final good, which will be defined below, $W(s^t)$ is the hourly wage in the home country, and $R(s^t)$ is the home-country rental rate of capital, where the prices of both factor inputs are measured in units of the final good. $P_H(s^t)$ is the price of the home intermediate good in the home country, $D(s^t)$ denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good (minus a small cost of holding bonds, $\Phi(\cdot)$) in period $t+1$ regardless of the state of nature, and $\overline{Q}(s^t)$ is its price, measured in units of the home intermediate good. The function $\Phi(\cdot)$ is an arbitrarily small cost of holding bonds measured in units of the home intermediate good.[9]

Following the existing literature, $\Phi(\cdot)$ takes the functional form

$$\Phi[D(s^t)]=\frac{\phi}{2}A(s^{t-1})\left[\frac{D(s^t)}{A(s^{t-1})}\right]^2, \tag{4}$$

where we have modified the adjustment cost function to ensure balanced growth.

3.2. Firms

We now describe the final and intermediate goods producers problems. In this subsection, we also the TFP process that it is the main departure from the standard literature.

3.2.1. Final goods producers

The final good in the home country, $Y(s^t)$, is produced using home intermediate goods, $Y_H(s^t)$, and foreign intermediate goods, $Y_F(s^t)$, with the following technology:

$$Y(s^t)=[\omega^{1/\theta}Y_H(s^t)^{(\theta-1)/\theta}+(1-\omega)^{1/\theta}Y_F(s^t)^{(\theta-1)/\theta}]^{\theta/(\theta-1)}, \tag{5}$$

where ω denotes the fraction of home intermediate goods that are used for the production of the home final good and θ controls the elasticity of substitution between home and foreign intermediate goods. Therefore, the representative final goods producer in the home country solves the following problem:

$$\max_{Y(s^t)\ge 0, Y_H(s^t)\ge 0, Y_F(s^t)\ge 0} P(s^t)Y(s^t)-P_H(s^t)Y_H(s^t)-P_F^*(s^t)Y_F(s^t), \tag{6}$$

subject to the production function (5), where $P_F^*(s^t)$ is the price of the foreign intermediate good in the home country.

3.2.2. Intermediate goods producers

The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves:

$$\max_{L(s^t)\ge 0, K(s^{t-1})\ge 0} P_H(s^t)Y_H(s^t)+P_H^*(s^t)Y_H^*(s^t)-P(s^t)[W(s^t)L(s^t)+R(s^t)K(s^{t-1})], \tag{7}$$

[9] The $\Phi(\cdot)$ cost is introduced to ensure stationarity of the level of $D(s^t)$ in IRBC models with incomplete markets, as discussed by Heathcote and Perri (2002). We choose the cost to be numerically small, so it does not affect the dynamics of the rest of the variables.

subject to the production function

$$Y_H(s^t)+Y_H^*(s^t)=A(s^t)^{1-\alpha}K(s^{t-1})^{\alpha}L(s^t)^{1-\alpha}, \tag{8}$$

where $Y_H(s^t)$ is the amount of home intermediate goods sold to the home final goods producers, $Y_H^*(s^t)$ is the amount of home intermediate goods sold to the foreign final goods producers, $A(s^t)$ is a stochastic process describing TFP of home intermediate goods producers, which is characterized below, and $P_H^*(s^t)$ is the price of the home intermediate good in the foreign country.

3.2.3. The processes for TFP

As mentioned above, the main departure from the standard model in the IRBC literature is the assumption that $\log A(s^t)$ and $\log A^*(s^t)$ are cointegrated of order C(1,1). This new assumption involves specifying the following VECM for the law of motion driving the log first-differences of TFP processes for both the home and the foreign country:

$$\begin{pmatrix} \Delta\log A(s^t) \\ \Delta\log A^*(s^t) \end{pmatrix} = \begin{pmatrix} c \\ c^* \end{pmatrix} + \begin{pmatrix} \kappa \\ \kappa^* \end{pmatrix}[\log A(s^{t-1})-\gamma\log A^*(s^{t-1})-\log\xi]+\begin{pmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{pmatrix}, \tag{9}$$

where $(1,-\gamma)$ is called the cointegrating vector, ξ is the constant in the cointegrating relationship, $\varepsilon(s^t)\sim N(0,\sigma)$ and $\varepsilon^*(s^t)\sim N(0,\sigma^*)$, $\varepsilon(s^t)$ and $\varepsilon^*(s^t)$ can be correlated, and Δ is the first-difference operator.[10]

This VECM representation implies that deviations of today's differences of TFP with respect to its mean value depend not only on lags of home and foreign differences of TFP but also on a function of the ratio of lagged home and foreign TFP, $A(s^{t-1})/[\xi A^*(s^{t-1})^{\gamma}]$. Thus, if the ratio $A(s^{t-1})/[\xi A^*(s^{t-1})^{\gamma}]$ is larger than its long-run value, then $\kappa<0$ and $\kappa^*>0$ will imply that $\Delta\log A(s^t)$ would fall and $\Delta\log A^*(s^t)$ would rise, driving both series toward a new common level. The VECM representation also implies that $\Delta\log A(s^t)$, $\Delta\log A^*(s^t)$, and $\log A(s^{t-1})-\gamma\log A^*(s^{t-1})-\log\xi$ are stationary processes.

3.3. Market clearing

The model is closed with the following market clearing conditions in the final goods markets

$$C(s^t)+X(s^t)=Y(s^t) \tag{10}$$

and in the international bond market

$$D(s^t)+D^*(s^t)=0. \tag{11}$$

3.4. Equilibrium

In this subsection, we first define equilibrium and then describe the set of equilibrium conditions that characterize equilibrium.

3.4.1. Equilibrium definition

Given the law of motion for TFP shocks defined by (9), an equilibrium for this economy is a set of allocations for home consumers, $C(s^t)$, $L(s^t)$, $X(s^t)$, $K(s^t)$, and $D(s^t)$; and foreign consumers, $C^*(s^t)$, $L^*(s^t)$, $X^*(s^t)$, $K^*(s^t)$, and $D^*(s^t)$; allocations for home and foreign intermediate goods producers, $Y_H(s^t)$, $Y_H^*(s^t)$, $Y_F(s^t)$ and $Y_F^*(s^t)$; allocations for home and foreign final goods producers, $Y(s^t)$ and $Y^*(s^t)$; intermediate goods prices $P_H(s^t)$, $P_H^*(s^t)$, $P_F(s^t)$ and $P_F^*(s^t)$; final goods prices $P(s^t)$ and $P^*(s^t)$; rental prices of labor and capital in the home and foreign country, $W(s^t)$, $R(s^t)$, $W^*(s^t)$, and $R^*(s^t)$ and the price of the bond $\overline{Q}(s^t)$ such that: (i) given prices, households' allocations solve the households' problem; (ii) given prices, intermediate goods producers' allocations solve the intermediate goods producers' problem; (iii) given prices, final goods producers allocations solve the final goods producers' problem; and (iv) markets clear.

3.4.2. Equilibrium conditions

At this point, it is useful to define the following relative prices: $\tilde{P}_H(s^t)=P_H(s^t)/P(s^t)$, $\tilde{P}_F^*(s^t)=P_F^*(s^t)/P^*(s^t)$ and $RER(s^t)=P^*(s^t)/P(s^t)$. Note that $\tilde{P}_H(s^t)$ is the price of home intermediate goods in terms of home final goods, $\tilde{P}_F^*(s^t)$ is the price of foreign intermediate goods in terms of foreign final goods, which appears in the foreign country's budget constraint, and $RER(s^t)$ is the RER between the home and foreign countries. The law of one price (LOP) holds; hence, $P_H(s^t)=P_H^*(s^t)$ and $P_F(s^t)=P_F^*(s^t)$.

The equilibrium conditions include the first order conditions of households, intermediate and final goods producers in both countries, as well as the relevant laws of motion, production functions, and market clearing conditions. We only present first order conditions for the home country because of symmetry. The marginal utility of consumption and the

[10] Here we restrict ourselves to a VECM with zero lags. This assumption is motivated by the empirical results to be presented in Section 4, where no lags are significant.

labor supply are given by

$$U_C(s^t)=\lambda(s^t), \tag{12}$$

$$\frac{U_L(s^t)}{U_C(s^t)}=W(s^t), \tag{13}$$

where U_x denotes the partial derivative of the utility function U with respect to variable x. The first order condition with respect to capital delivers an intertemporal condition that relates the marginal rate of consumption to the rental rate of capital and the depreciation rate:

$$\lambda(s^t)=\beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\lambda(s^{t+1})[R(s^{t+1})+1-\delta], \tag{14}$$

where $\pi(s^{t+1}|s^t)=\pi(s^{t+1})/\pi(s^t)$ is the conditional probability of s^{t+1} given s^t. The law of motion of home capital is

$$K(s^t)=(1-\delta)K(s^{t-1})+X(s^t), \tag{15}$$

the analogous expressions for the foreign country are omitted.

The optimal choice by households of the home country delivers the following expression for the price of the riskless bond:

$$\overline{Q}(s^t)=\beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\frac{\lambda(s^{t+1})}{\lambda(s^t)}\frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)}-\frac{\Phi'[D(s^t)]}{\beta}. \tag{16}$$

The next condition equates the price of the riskless bond to the cost of adjusting bonds:

$$\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\left[\frac{\lambda^*(s^{t+1})}{\lambda^*(s^t)}\frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)}\frac{RER(s^t)}{RER(s^{t+1})}-\frac{\lambda(s^{t+1})}{\lambda(s^t)}\frac{\tilde{P}_H(s^{t+1})}{\tilde{P}_H(s^t)}\right]=-\frac{\Phi'[D(s^t)]}{\beta}. \tag{17}$$

From the intermediate goods producers' maximization problems, labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

$$W(s^t)=(1-\alpha)\tilde{P}_H(s^t)A(s^t)^{1-\alpha}K(s^{t-1})^{\alpha}L(s^t)^{-\alpha}, \tag{18}$$

$$R(s^t)=\alpha\tilde{P}_H(s^t)A(s^t)^{1-\alpha}K(s^{t-1})^{\alpha-1}L(s^t)^{1-\alpha}. \tag{19}$$

From the final goods producers' maximization problem, the demands of intermediate goods depend on their relative price:

$$Y_H(s^t)=\omega\tilde{P}_H(s^t)^{-\theta}Y(s^t), \tag{20}$$

$$Y_F(s^t)=(1-\omega)(\tilde{P}_F^*(s^t)RER(s^t))^{-\theta}Y(s^t). \tag{21}$$

Using the production functions of the final goods

$$Y(s^t)=[\omega^{1/\theta}Y_H(s^t)^{(\theta-1)/\theta}+(1-\omega)^{1/\theta}Y_F(s^t)^{(\theta-1)/\theta}]^{\theta/(\theta-1)} \tag{22}$$

and the demand equations for intermediate goods just described, the final goods deflator in the home country is

$$P(s^t)=[\omega P_H(s^t)^{1-\theta}+(1-\omega)P_F(s^t)^{1-\theta}]^{1/(1-\theta)}. \tag{23}$$

Hence, given that the LOP holds, the RER is equal to

$$RER(s^t)=\frac{P^*(s^t)}{P(s^t)}=\frac{[\omega P_F(s^t)^{1-\theta}+(1-\omega)P_H(s^t)^{1-\theta}]^{1/(1-\theta)}}{[\omega P_H(s^t)^{1-\theta}+(1-\omega)P_F(s^t)^{1-\theta}]^{1/(1-\theta)}}. \tag{24}$$

Note that the only source of RER fluctuations is the presence of home bias ($\omega>1/2$). Also, intermediate goods, final goods, and bond markets clear as in Eqs. (8), (10), and (11). Finally, the law of motion of the level of debt

$$\tilde{P}_H(s^t)\overline{Q}(s^t)D(s^t)=\tilde{P}_H(s^t)Y_H^*(s^t)-\tilde{P}_F^*(s^t)RER(s^t)Y_F(s^t)+\tilde{P}_H(s^t)D(s^{t-1})-\tilde{P}_H(s^t)\Phi[D(s^t)], \tag{25}$$

is obtained using (2) and the fact that intermediate and final goods producers at home make zero profits. Finally, the productivity shocks follow the VECM described in Section 3.2.3.

3.5. Balanced growth and the restriction on the cointegrating vector

Eqs. (8), (10) and (11), together with (12)–(25) and their foreign-country counterparts, and the VECM process for TFP characterize the equilibrium in this model. Since both $\log A(s^t)$ and $\log A^*(s^t)$ are integrated processes, it is necessary to normalize the equilibrium conditions in order to obtain a stationary system more amenable to study. Following King et al. (1988), home-country variables that have a trend are normalized by the lagged domestic level of TFP, $A(s^{t-1})$, and the foreign-country variables that have a trend are normalized by the lagged foreign level of TFP, $A^*(s^{t-1})$. In the online appendix, the full set of normalized equilibrium conditions is presented.

The model requires some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King et al. (1988) are sufficient for the existence of balanced growth in a closed economy real business cycle (RBC) model. However, a two-country model requires an additional restriction on the cointegrating vector to ensure balanced growth. In particular, the ratio $A(s^{t-1})/A^*(s^{t-1})$ must be stationary.

In order to understand why the international dimension of the model requires this additional restriction, consider the normalized demand of imported foreign-produced intermediate goods by the home country:

$$\hat{Y}_F(s^t) = (1-\omega)[\tilde{P}_F^*(s^t)RER(s^t)]^{-\theta}\hat{Y}(s^t)\frac{A(s^{t-1})}{A^*(s^{t-1})}, \tag{26}$$

where $\hat{Y}_F(s^t) = Y_F(s^t)/A^*(s^{t-1})$ while $\hat{Y}(s^t) = Y(s^t)/A(s^{t-1})$. Since $\tilde{P}_F^*(s^t)$ and $RER(s^t)$ are stationary, if the ratio between $A(s^{t-1})$ and $A^*(s^{t-1})$ was to be non-stationary, the ratio between $\hat{Y}_F(s^t)$ and $\hat{Y}(s^t)$ would also be non-stationary and balanced growth would not exist. A similar argument holds for other normalized equilibrium conditions.

Our VECM implies that the ratio between $A(s^{t-1})$ and $A^*(s^{t-1})^\gamma$ is stationary. Therefore, a sufficient condition for balanced growth is that the parameter γ equals one or, equivalently, that the cointegrating vector equals $(1,-1)$.

4. Estimation of the VECM

This section describes the constructed TFP series for the U.S. and the R.W., and presents two important results. First, the assumption that the TFP processes are cointegrated of order C(1,1) cannot be rejected in the data. By the Granger representation theorem this implies that the VECM specification is valid. Second, the restriction imposed by balanced growth, i.e., that the parameter γ is equal to one, also cannot be rejected in the data. To conclude, the VECM is estimated, and the parameter values are used to simulate the model in Section 5.

4.1. Data

For the U.S., quarterly real GDP data are obtained from the Bureau of Economic Analysis and hours and employment data from the Organization for Economic Cooperation and Development (OECD). Real capital stock data are also obtained from the OECD database. The R.W. aggregate is the Euro area plus the United Kingdom, Canada, Japan, and Australia. This group accounts for about 50 percent of the basket of currencies that the Federal Reserve uses to construct the RER for the U.S. dollar. For all countries except the Euro area nominal GDP, hours, employment, and real capital stock are obtained from the OECD. For the Euro area, the data source for nominal GDP, employment, and real capital stock is the area wide model (AWM). Hours are as reported in Christoffel et al. (2009). The sample period begins at 1973:1 and ends at 2006:4, which is when the hours series for the Euro area ends. Ideally, one would want to include additional countries that represent an important and increasing share of trade with the U.S., such as China and other emerging countries, but long quarterly series are not available.

Nominal GDPs of the R.W. are aggregated using PPP nominal exchange rates to convert each national nominal output to current U.S. dollars, and then deflated using the GDP deflator of the U.S. (base year 2000) to obtain the aggregate R.W. real GDP. Aggregate R.W. hours series is constructed by aggregating the number of employees times hours per employee for each country. Real capital stocks series in domestic currency (base year 2000) are aggregated using the base year 2000 PPP RERs. Then, the TFP processes are constructed as follows:

$$\log A(s^t) = \frac{\log Y(s^t) - (1-\alpha)\log L(s^t) - \alpha \log K(s^{t-1})}{1-\alpha}, \tag{27}$$

$$\log A^*(s^t) = \frac{\log Y^*(s^t) - (1-\alpha)\log L^*(s^t) - \alpha \log K^*(s^{t-1})}{1-\alpha}, \tag{28}$$

where α is the capital share of output and takes a value of 0.36. Backus et al. (1992) and Heathcote and Perri (2002, 2009) use a similar approach when constructing TFP series for the U.S. and a R.W. aggregate but ignore capital dynamics. Given the focus on long-run properties of the model, capital stock is important for the analysis.

4.2. Integration and cointegration properties

This section presents evidence supporting the assumption that the TFP processes for the U.S. and the R.W. are cointegrated of order C(1,1). After providing empirical support for the presence of one unit root in each univariate processes, the Johansen (1991) procedure is applied to test for cointegration. Both the trace and the maximum eigenvalue methods support the existence of a cointegrating vector.

Univariate analysis of the TFP processes for the U.S. and the R.W. strongly indicates that both series can be characterized by unit root processes with drift. Table 2 presents results for the U.S. TFP process using the following

Table 2
Unit root tests.

Method	log TFP U.S.		log TFP R.W.	
	Level statistic	First-diff. statistic	Level statistic	First-diff. statistic
ADF	−2.96*	−11.57	−1.25*	−9.35
DF-GLS	−1.94*	−11.18	−0.21*	−5.05
P_T-GLS	23.74*	1.61	123.18*	3.05
MZ_α	−4.96*	−84.20	−0.37*	−14.70
MZ_t	−1.50*	−6.48	−0.23*	−2.48
MS_B	0.30*	0.07	0.63*	0.16**

Notes: ADF stands for augmented Dickey–Fuller test. DF-GLS stands for Elliott–Rothenberg–Stock detrended residuals test statistic. P_T–GLS stands for Elliott-Rothenberg-Stock point-optimal test statistic. MZ_α, MZ_t, and MS_B stand for the class of modified tests analyzed in Ng and Perron (2001). For the ADF and DF-GLS, present *t*-statistics are shown. For P_T–GLS *P*-statistics are presented, while for the MZ_α, MZ_t, and MS_B the Ng–Perron test statistics are shown. * denotes null hypothesis of unit root not rejected at 5 percent level. ** denotes null hypothesis of unit root not rejected at 5 percent level but rejected at 10 percent.

Table 3
Cointegration statistics II: Johansen's test.

Number of vectors	Eigenvalue	Trace	*p*-Value	Max-Eigenvalue	*p*-Value
0	0.14	24.93	0.001	21.52	0.003
1	0.02	3.86	0.07	3.84	0.07

Notes: This table reports cointegration tests based on the eigenvalue and trace statistics of the Johansen maximum likelihood procedure.

Table 4
Likelihood ratio tests.

Restriction	Likelihood value	Degrees of freedom	*p*-Value
None	992.88	–	–
$\gamma = 1$	992.88	1	0.96
$\kappa = -\kappa^*$	992.3	2	0.57

Notes: This table reports the likelihood ratio test for different restrictions on the parameters of the VECM model. The first restrictions relate to the value of the cointegration vector and the second one to the symmetry of the speeds of convergence.

commonly applied unit root tests: augmented Dickey–Fuller; the DF-GLS and the optimal point statistic ($P_T GLS$), both of Elliott et al. (1996); and the modified MZ_α, MZ_t, and MS_B of Ng and Perron (2001). The lag length is chosen using the Schwarz information criterion. In each case a constant and a trend are included in the specification. Table 2 also presents the same unit root test results for the R.W. TFP process. None of the test statistics are close to rejecting the null hypothesis of unit root at the 5 percent critical value. Using the same statistics, unit root tests on the first-difference of the TFP processes for the U.S. and the R.W. are stationary. For the U.S. all the tests reject the null hypothesis of unit root at the 5 percent critical value. For the R.W. all the tests reject the null hypothesis of unit root at the 5 percent critical value except the MS_B test which rejects it at the 10 percent value.

Turning to the question of cointegration, if $\log A(s^t)$ and $\log A^*(s^t)$ share one common stochastic trend (balanced growth), an estimated VAR must possess a single eigenvalue equal to one and all other eigenvalues have to be less than one. To check this possibility an unrestricted VAR with one lag and a deterministic trend for the two-variables system $[\log A(s^t), \log A^*(s^t)]$ is estimated, where the number of lags was chosen using the Schwarz information criterion. The highest eigenvalue equals 0.99, while the second highest is 0.95. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). The cointegration test assumes a linear trend and a constant in the cointegrating vector. The data strongly support a single eigenvalue.

4.3. The VECM model

To conclude the empirical analysis of the joint behavior of TFP across countries, two additional important empirical results are discussed. First, the null hypothesis that $\gamma = 1$ cannot be rejected by the data using a likelihood ratio test. This rejection is very important because a cointegrating vector $(1, -1)$ implies that the balanced growth path hypothesis cannot be rejected. Next, a likelihood ratio test provides evidence supporting the null hypothesis that the coefficients related to the speed of adjustment in the cointegrating vector are equal and of opposite sign, i.e., $\kappa = -\kappa^*$. Table 4 reports the outcome of the two likelihood ratio tests. The last row presents the joint test of the two restrictions.

Table 5
VECM model.

c	c^*	κ
0.001**	0.006*	−0.007*
(1.76)	(12.46)	(−4.19)

Notes: t-statistics in parenthesis. * denotes significance at the 5 percent level and ** denotes significance at the 10 percent level.

The estimated restricted model delivers the parameter estimates reported in Table 5. The restricted VECM includes with zero lags. It is worth noting that the coefficient of the speed of adjustment, while significant, is quantitatively small so TFP processes converge slowly over time. A low speed of adjustment parameter (κ) implies slow spillover of TFP shocks across countries. This feature is key to explain the results of the present paper. The constant terms c and c^* have different point estimates. However, this difference does not imply that the growth rates of both TFP processes are different. Indeed, because the cointegrating vector is $(1,-1)$, they must grow at the same rate along the balanced growth path. Given these parameter estimates, the implied long-run growth rate of TFP processes is 1.44 percent (in annualized terms).

The estimated standard deviation of the innovations σ and σ^* is 0.0105 and 0.0088, respectively. In the model simulations, the correlation between $\varepsilon(s^t)$ and $\varepsilon^*(s^t)$ is set to zero, since the null hypothesis of no correlation could not be rejected in the data.

5. Results

In this section, we describe the results. We start by discussing the parametrization. Then, we analyze how the model matches the volatility of the RER and the intuition behind the results. We finalize by illustrating how the model explains the increase in RER volatility observed in the data during the last two decades.

5.1. Parameterization

The baseline parameterization closely follows that in Heathcote and Perri (2002). The discount factor β is set equal to 0.99, which implies an annual rate of return on capital of 4 percent. The consumption share, τ, is set to 0.34; the coefficient of risk aversion, σ, is set to 2 as in Backus et al. (1992). We assume a cost of bond holdings, ϕ, of one basis point. Parameters on technology are standard in the literature: the depreciation rate, δ, is set to a quarterly value of 0.025, the capital share of output is set to $\alpha=0.36$, and home bias for domestic intermediate goods is set to $\omega=0.9$, which implies the observed import/output ratio in the steady state. Two possible values for the elasticity of substitution between intermediate goods are considered, $\theta=0.85$ and 0.62. The first value is based on Heathcote and Perri (2002); the second value is a bit higher than the lower bound of 0.5 considered by Corsetti et al. (2008b). The VECM is calibrated as described in Table 5. In most cases, the results of this calibration are compared to the ones obtained when using stationary TFP shocks. For the stationary case, the parameters of the VAR(1) process for TFP shocks are calibrated as in Heathcote and Perri (2002):

$$a_t=\rho a_{t-1}+v a^*_{t-1}+\varepsilon_t \tag{29}$$

and

$$a^*_t=\rho a^*_{t-1}+v a_{t-1}+\varepsilon^*_t \tag{30}$$

where $a_t=\log A_t$, $a_t=\log A^*_t$, $\rho=0.97$, $v=0.025$, $Var(\varepsilon_t)=Var(\varepsilon^*_t)=(0.0073)^2$, and $corr(\varepsilon_t,\varepsilon^*_t)=0.29$.

5.2. Matching RER volatility

Since the model is non-stationary, we found it convenient to compute HP-filtered moments by stochastic simulation. Hence, series of TFP shocks are drawn based on the empirical estimates and then used to simulate the model. The normalized model is solved taking a log-linear approximation around the steady state. To avoid dependence on initial values, the initial 1000 periods of the simulation are discarded, and the remaining 125 periods are used to compute statistics. The HP-filter is applied to the relevant series from the model (output, consumption, investment, employment, and the RER) and second moments are computed from the filtered series. This procedure is repeated 5000 times. Table 6 reports the average of the simulations.

The first and second rows of Table 6 report the results of the economy with cointegrated TFP and high and low values for the trade elasticity, θ, respectively. The next two rows show the results for the stationary model. Overall, models with cointegrated shocks generate higher relative volatility of the RER with respect to output than models with stationary shocks. Note that with high trade elasticity and cointegrated shocks, the relative volatility of the RER more than doubles with respect to the model with stationary shocks (1.31 versus 0.75). Hence the model improves, from explaining less than 25 percent of the observed relative volatility of the RER to explaining more than 40 percent.

Table 6
Full sample results.

	$SD(Y)$	$RSD(C)$	$RSD(X)$	$RSD(N)$	$RSD(RER)$	$\rho(RER)$
Data	1.58	0.76	4.55	0.75	3.06	0.82
Coint., $\theta=0.85$	0.93	0.65	2.31	0.29	1.31	0.72
Coint., $\theta=0.62$	0.85	0.66	2.46	0.27	3.13	0.70
Stat., $\theta=0.85$	1.19	0.52	2.53	0.32	0.75	0.77
Stat., $\theta=0.62$	1.12	0.54	2.51	0.31	1.41	0.75

	Correlations			
	(Y,N)	(Y,C)	(Y,X)	$(RER,C/C^*)$
Data	0.87	0.84	0.91	-0.04
Coint., $\theta=0.85$	0.94	0.95	0.97	0.95
Coint., $\theta=0.62$	0.97	0.98	0.98	0.97
Stat., $\theta=0.85$	0.97	0.93	0.97	0.99
Stat., $\theta=0.62$	0.97	0.93	0.97	0.99
	(Y,Y*)	(C,C*)	(X,X*)	(N,N*)
Data	0.44	0.36	0.28	0.40
Coint., $\theta=0.85$	0.11	0.45	−0.21	−0.29
Coint., $\theta=0.62$	0.48	0.69	0.15	0.13
Stat., $\theta=0.85$	0.18	0.70	−0.18	−0.21
Stat., $\theta=0.62$	0.33	0.81	−0.05	−0.05

Notes: SD denotes standard deviation of HP-filtered series. RSD denotes standard deviation of HP-filtered series relative to HP-filtered output. ρ denotes first autocorrelation. Where * denotes the R.W.

As expected, for lower values of the trade elasticity, the relative volatility of the RER increases under both the stationary and cointegrated models. The striking finding is that the model with cointegrated shocks and elasticity equal to 0.62 is able to closely match the relative volatility of the RER (3.13 in the model versus 3.06 in the data), while the model with stationary shocks and the same elasticity produces RER volatility of just 1.41 (which represents only about 40 percent of the fluctuations in the data). Interestingly, even though the model with cointegrated shocks improves significantly in matching the RER volatility, it does not affect the fit of other unconditional moments. Both the stationary and the cointegrated shocks models display very similar volatilities of consumption, hours, and investment relative to output. Also, both models display similar cross-correlations between consumption, hours, and investment relative to output and autocorrelations of RERs and neither of the models can explain the Backus and Smith (1993) puzzle.

All models produce similar cross-correlations between U.S. and R.W. for output and consumption, but the model with cointegrated shocks and $\theta=0.62$ can better explain the international co-movement of investment and hours. For the case of $\theta=0.62$, the stationary model produces a cross-correlation between domestic and foreign output of 0.33, while with cointegrated shocks is 0.48 and the observed correlation is 0.44. For the case of consumption the stationary and cointegrated models produce cross-country correlations of 0.81 and 0.69, respectively, while the observed correlation is 0.36. For investment, the numbers are -0.05, 0.15 and 0.28, respectively. Finally, for hours, the stationary and cointegrated shock models produce a correlation of -0.05 and 0.13, respectively, while the observed correlation is 0.40. Unfortunately, the model with cointegrated shocks, like the model with stationary shocks, cannot solve the "quantity puzzle". In the data, output is more correlated than consumption across countries, while, in the model, consumption is more correlated than output. In any case, the cointegrated model with $\theta=0.62$ does better than the other versions.

Although not reported in the tables, the model was also simulated with two alternative asset market structures: complete markets and financial autarky. In the first case, agents have access to a full set of state-contingent bonds that pay one unit of the domestic intermediate good in every state of the world. In the second case, the cost of holding bonds, $\Phi'[D(s^t)]$, is calibrated to a very large number such that intertemporal trade never occurs. As expected (see Heathcote and Perri, 2002), the version of the model with complete markets generates lower relative volatility of the RER (falling from 1.31 to 0.90 when $\theta=0.85$, and from 3.13 to 1.01 when $\theta=0.62$), while the version of the model with financial autarky delivers a larger relative volatility of the RER (increasing from 1.31 to 1.51 when $\theta=0.85$, and from 3.13 to 4.14 when $\theta=0.62$). Therefore, the presence of incomplete markets helps the model with cointegrated shocks in increasing the relative volatility of RER (at least with respect to the complete markets case).

Finally, it is important to recognize that RERs in the data differ in two important ways from the theory. First, existing empirical evidence shows deviations from LOP, which the model ignores. With respect to this point, Crucini and Shintani (2008) argue that LOP deviations lack persistence at the microeconomic level, which may help justify abstracting from LOP deviations in the context of the present study. As a rough check on this, a version of the model with Calvo-type sticky prices and domestic currency pricing was simulated, to find that the results do not change significantly. In any case, it is important to mention that ignoring the deviations makes it more difficult for this model to match the data: deviations from LOP are one important source of the fluctuations in the data.

Second, the measure of RER constructed in this paper uses trade weights applied to CPIs. CPIs include non-traded goods, which are excluded in the model. Thus, there is the issue of whether the CPI-based RER is the right empirical counterpart to the model-based RER: there is a gap between theory and measurement. To try to fill the gap, two alternative measures of the RER were constructed. The first measure uses PPIs instead of CPIs. The idea, going back to Engel (1999), is that CPIs have a larger share of non-traded goods in the basket than the PPIs, and hence the PPI is a better measure of tradable goods price index. The second measure uses export deflators as the relevant tradable goods price indices, since it measures the prices of goods that are actually shipped internationally.[11]

The volatility of the PPI- and CPI-based RERs is similar, which reinforces Engel's (1999) result. In particular, for the period 1973:1 to 2006:4 the standard deviation of the PPI-based RER is 2.96 larger than the standard deviation of output while using the CPI-based RER, the standard deviation of RER is 3.06 larger than the standard deviation of output (as shown in Table 6). For the same period, the standard deviation of the export deflator-based RER is 2.34 larger than the standard deviation of output. Hence, using any of these alternative measures does not change the main message of the paper.

5.3. Intuition

In the model, four key parameters drive the behavior of the volatility of the RER with respect to output: (i) the elasticity of substitution between home and foreign goods, (ii) the fraction of intermediate goods in the production of the final good (or "home bias"), (iii) the persistence of the TFP process, and (iv) the persistence in the differential of TFP processes across countries. This subsection studies how these four elements shape the results. The first two components are well-known in the IRBC literature, so a brief discussion is presented here. The second two components are new to this paper, so more space is devoted to them.

5.3.1. The role of the elasticity of substitution and home bias

It is important to note that since the model only includes tradable intermediate goods, the RER relates to input prices as follows:

$$RER(s^t) = \frac{[\omega P_F(s^t)^{1-\theta} + (1-\omega)P_H(s^t)^{1-\theta}]^{1/(1-\theta)}}{[\omega P_H(s^t)^{1-\theta} + (1-\omega)P_F(s^t)^{1-\theta}]^{1/(1-\theta)}}. \tag{31}$$

Thus, for a given volatility of the relative prices of intermediate goods, the model needs low elasticity of substitution (low θ) and/or high home bias (high ω) in order to match the observed RER volatility. However, this is largely an artifact of ignoring non-traded intermediate goods. An alternative would be introducing a non-traded intermediate good as a substitute to the asymmetric aggregate among traded goods. However, there are challenges associated with measuring TFP in the tradable and non-traded sectors in each of the two countries at quarterly frequency. Since one of the key ingredients of this paper is, indeed, to properly measure TFP, it is not feasible to follow this route here.

In the model, it is difficult to obtain one analytical expression linking the volatility of the RER to output. However, by combining the two demand equations for home and foreign intermediate goods (20) and (21), normalizing by TFP in each country, and log-linearizing, the following expression arises:

$$\hat{y}_{H,t} - \hat{y}_{F,t} = \frac{\theta}{2\omega-1} rer_t - (a_{t-1} - a^*_{t-1}). \tag{32}$$

In words: for a given volatility of relative quantities and relative TFP, lowering θ or increasing ω would imply higher RER fluctuations (see also Backus et al., 1994).

Of course, this is not a full argument, since changes in θ and/or ω will affect all variables in general equilibrium. The full general equilibrium effects are elaborated in Fig. 2, which compute the ratio between the standard deviation of the RER with respect to the standard deviation of output for different combinations of θ and ω. As expected, increasing home bias (ω) or decreasing the elasticity of substitution (θ) leads to higher relative volatility of the RER.

5.3.2. The role of the estimated coefficients of TFP processes

The key fact of productivity in Baxter and Crucini (1995), at least in terms of the relevance of asset market restrictions they explored, is not the persistence of TFP processes but the persistence of the productivity gap. The reason is clear from their Hicksian decompositions: home and foreign wealth effects, due to incomplete markets, arise when productivity diverges across countries in a persistent fashion.

The joint process of TFP shocks across countries can be written as follows:

$$a_t = \rho a_{t-1} + \nu a^*_{t-1} + \varepsilon_t, \tag{33}$$

$$a^*_t = \rho a^*_{t-1} + \nu a_{t-1} + \varepsilon^*_t, \tag{34}$$

[11] We construct the export deflator-based real exchange rate series between 1973 and 2009. We obtain export deflators series for Australia, Canada, Japan, the United Kingdom and the U.S. using national data sources, while data for the Euro area come from the AWM.

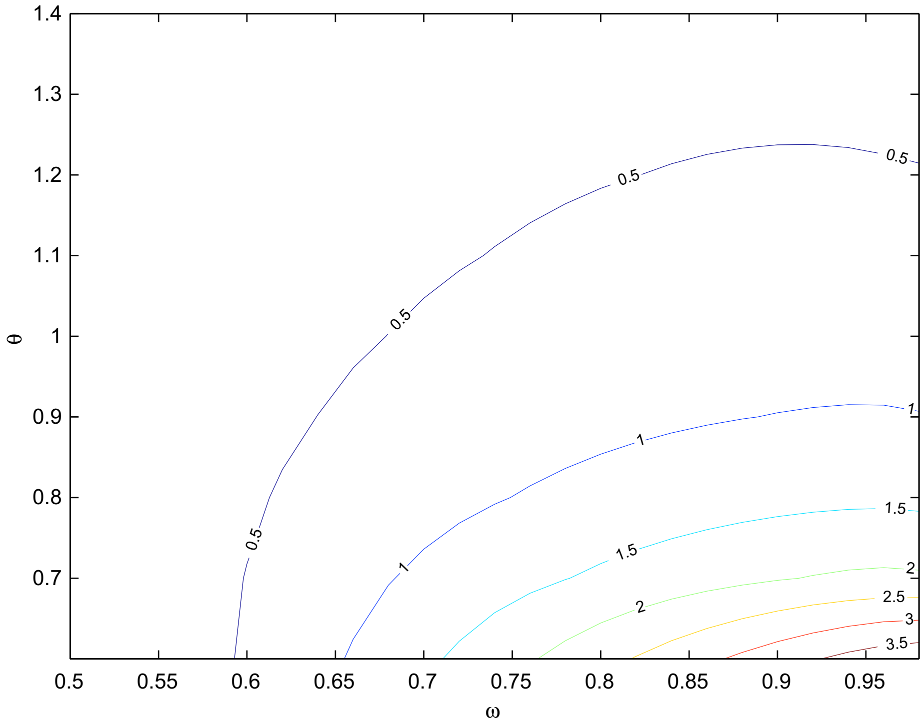

Fig. 2. Standard deviation of the RER with respect to the standard deviation of output when θ and ω change.

as in Backus et al. (1994) and Heathcote and Perri (2002). The level of TFP in a country is related to its own lag with coefficient ρ, and to the lag of the other country's TFP with coefficient v. The coefficient v is also known as the TFP spillover from one country to the other. Note that when $\rho=(1+\kappa)$ and $v=-\kappa$, this VAR equals the VECM with cointegrating vector $(1,-1)$ and symmetric convergence speed to the cointegrating relationship that is considered throughout this paper.[12]

Given that the two countries are of the same size, define world TFP as $a_t^w=a_t+a_t^*$, and relative TFP as $a_t^r=a_t-a_t^*$. These processes evolve as

$$a_t^w=(\rho+v)a_{t-1}^w+\varepsilon_t^w, \tag{35}$$

$$a_t^r=(\rho-v)a_{t-1}^r+\varepsilon_t^r, \tag{36}$$

where $\varepsilon_t^w=\varepsilon_t+\varepsilon_t^*$ and $\varepsilon_t^r=\varepsilon_t-\varepsilon_t^*$. The key parameters in the model are $(\rho+v)$, the persistence of the world TFP shock, and $(\rho-v)$, the persistence of relative TFP. In the VECM, $(\rho+v)=1$ and $\rho-v=1-2\kappa$. Therefore, given a common unit root in the system, a slower convergence to the cointegrating relationship (smaller κ) implies a higher persistence of relative TFP (higher $\rho-v$).

Fig. 3 shows the relative standard deviation of the RER as a function of $(\rho+v)$ and $(\rho-v)$. The mechanism works as follows. When a positive TFP shock hits the home economy, output, consumption, investment and hours worked increase in the home economy, and the real exchange rate depreciates. In the foreign country, output, investment and hours decrease and consumption increases because foreign households anticipate the arrival of the technology improvement in the future and react to the wealth effect.

Increasing the persistence of the world TFP shock increases the volatility of output but not to a large extent. However, for a given level of persistence in the world TFP shock, an increase in the persistence of the TFP differential reduces the standard deviation of output. Why? One the one hand, it delays the arrival of the TFP improvement in the foreign country and eases the wealth effect for the foreign economy. On the other hand, it increases the persistence of the TFP shock in the home country, intensifying the wealth effect. Hence, labor, investment and output respond less strongly in both countries reducing output volatility.

What happens with the volatility of the RER? The intuition is clear when changes in ρ and v are studied in isolation. When ρ increases but v remains fixed, the wealth effect leads home households to demand more consumption goods. In order to produce more final goods, the home country demands more intermediate goods from the foreign country, which, provided that the elasticity of substitution is low enough, leads to larger RER depreciations. Hence, the volatility of the RER increases. When ρ is fixed but v increases, the exact opposite occurs. In this case the wealth effect hits the foreign-country

[12] We are thankful to associate editor Mario Crucini for suggesting this presentation.

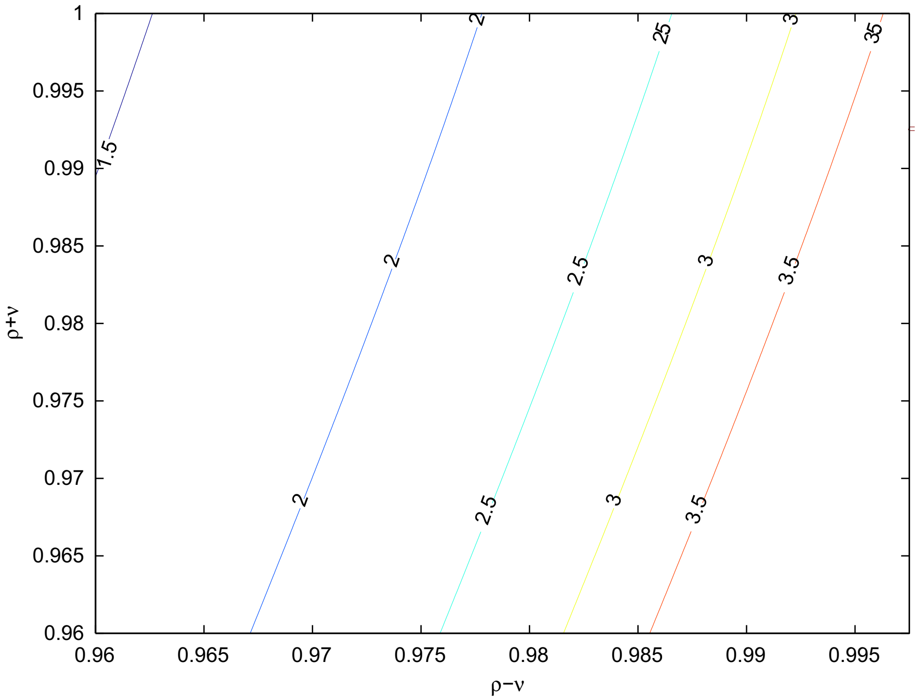

Fig. 3. Standard deviation of the RER with respect to the standard deviation of output when ρ and v change.

households. They know that productivity will transit positively to their country sooner and they demand more consumption goods than they would if spillovers were slower. Thus, the demand for home intermediate goods increases because foreign final goods producers substitute away from their intermediate goods. As a consequence, the price of home goods increases relative to that of foreign goods and the RER depreciates less than in a model with slower (or no) spillovers.

Therefore, an increase in world TFP $(\rho+v)$ holding constant the persistence of relative TFP $(\rho-v)$ means that both ρ and v increase. As discussed, these two coefficients have opposite effects on the volatility of the RER and, on net, increasing $(\rho+v)$ leads to a decline in the relative volatility of the RER. On the other hand, an increase in the persistence of relative TFP differential $(\rho-v)$ holding constant the persistence of world TFP shock $(\rho+v)$ means that ρ increases by as much as v decreases, which unequivocally increases the relative RER volatility.

In Fig. 3 it is possible to analyze the case that has been examined throughout the paper, that is, a VECM with cointegrating vector $(1,-1)$ and symmetric speed of convergence κ. This case involves horizontal movements along the line when $\rho+v=1$. Decreasing κ (i.e. increasing $\rho-v$ while keeping $\rho+v=1$) leads to a decline in the volatility of output, an increase in the relative volatility of the RER that matches the one observed in the data.[13]

5.4. Matching the increase in RER volatility

As described in Section 2, the volatility of the RER with respect to the volatility of output has increased in the last two decades in the U.S. As shown by the Quandt–Andrews test, the increase seems to occur at about 1993:4. As Table 6 shows the volatility of the RER has gone from less than three times the volatility of output during the period 1973:1 to 1993:4 to more than five times during the period 1994:1 to 2006:4. Using U.S. and R.W. data, this section presents evidence that relates the decrease in the speed of convergence to the cointegrating relationship, i.e., lower κ, with the increase in the relative volatility of the RER.

To formalize this, the VECM is estimated for two non-overlapping sub-samples.[14] The first sample goes from 1973:1 to 1993:4, while the second sub-sample goes from 1994:1 to 2006:4. We split the sample to match the results of the Quandt–Andrews test. For the first sub-sample the estimated value of the speed of adjustment term is larger in absolute value than the value estimated with the entire sample. In particular, κ moves from -0.007 to -0.008. Also, the standard deviation of

[13] Note that, theoretically, it would be possible to calibrate a stationary model with low persistence in world productivity and high persistence in the relative productivity that implies high RER volatility. However, this would require that $\rho-v>\rho+v$ and hence that v is negative. None of the available estimates suggest that spillovers are negative, indicating that this theoretical result is not supported by empirical evidence.

[14] We assume that the cointegrating relationship is the same across samples.

Table 7
Sub-sample results.

	$SD(Y)$	$RSD(C)$	$RSD(X)$	$RSD(N)$	$RSD(RER)$	$\rho(RER)$
1973–1993						
Data	1.89	0.78	4.47	0.79	2.72	0.82
Coint., $\theta = 0.85$	1.12	0.59	2.25	0.27	1.32	0.72
Coint., $\theta = 0.62$	1.01	0.62	2.17	0.25	2.85	0.71
1994–2006						
Data	0.88	0.78	4.82	0.90	5.01	0.82
Coint., $\theta = 0.85$	0.79	0.55	2.74	0.38	1.45	0.71
Coint., $\theta = 0.62$	0.73	0.66	2.01	0.42	4.31	0.72

Notes: SD denotes standard deviation of HP-filtered series. RSD denotes standard deviation of HP-filtered series relative to HP-filtered output. ρ denotes first autocorrelation.

the stochastic process for the U.S., σ, is estimated to be 0.012, while the standard deviation for the R.W., σ^*, is estimated to be 0.011. Both values are larger than the ones obtained when the whole sample is used.

In the second sub-sample, 1994:1 to 2006:4, the estimated speed of adjustment coefficient decreases dramatically with respect to both the full sample and the first sub-sample: the point estimate is -0.002. This means that the catching up process is much slower in the second part of the sample. This result indicates that the co-movement between TFPs in the post-1994 period is characterized by a very slow return to the long-run level. Finally, the standard deviations σ and σ^* are estimated to be 0.009 and 0.007, respectively. The sub-sample estimates of σ and σ^* reflect both the sample period and the countries in the R.W. aggregate. The large drop in σ and σ^* across sub-samples reveals the reduction in output volatility that the U.S. and other countries experienced after the 1980s (see Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000).

Table 7 reports the results. It is important to point out that the change in VECM parameters is entirely unexpected and then fully understood by the agents in the model. The results indicate that the change in the estimates of the VECM across samples is an important force behind the increase in the relative volatility of the RER. In the data the relative volatility of the RER increases by 80 percent across samples. Our simulations show that the model generates increases in relative volatility of around 50 percent for both low and high values of θ: the model can explain more than 60 percent of the increase in RER volatility.

6. Concluding remarks

This paper documents two empirical facts. First, that TFP processes of the U.S. and the R.W. are cointegrated with cointegrating vector $(1,-1)$ and, second, that the relative volatility of the RER with respect to output has increased in the U.S. during the last 20 years. Then, the paper has shown that introducing cointegrated TFP processes in an otherwise standard IRBC model increases the model's ability to explain RER volatility, without affecting the fit to other second moments of the data (and sometimes providing small improvements of fit). If one allows the speed of convergence to the cointegrating vector to change as it does in the data, the model can also explain the observed increase in the relative volatility of the RER.

For future research, it would be interesting to introduce cointegrated TFP processes in medium-scale open economy macroeconomic models, which typically include more frictions, and try to match a larger set of domestic and international variables (see Adolfson et al., 2007). Also, instead of analyzing the volatility of the RER between the U.S. and a synthetically constructed R.W., one could compute bilateral RERs and relate them to bilateral trade flows and output co-movements. This exercise is beyond the scope of this paper, but it could be an interesting line of research given that the equilibrium trade literature is consistently finding that RERs and gross bilateral trade flows are closely related.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jmoneco.2011.03.005.

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